

SENIOR SIX HOLIDAY PACKAGE/**INSTRUCTIONS:****Attempt all questions in their respective order****Geometrical instruments and silent non-programmable calculators may be used.**

Q1. Find the matrix X such that $2X + 3A = B$ if $A = \begin{pmatrix} -2 & -1 \\ 1 & 0 \\ 3 & -4 \end{pmatrix}$ $B = \begin{pmatrix} 0 & 3 \\ 2 & 0 \\ -4 & -1 \end{pmatrix}$ / **4marks**

Q2. The lengths of sides of a triangle are 7m, 10m and 16m. Calculate the size of each angle of the triangle. Give the answer correct to the nearest degree. / **4 marks**

Q3. You are given the system of equation: $\begin{cases} kx - 9y = -3 \\ 4x + (k - 12)y = k \end{cases}$ Where k is a real number.

For which value of k does the system of equation have no solution? / **5 marks**

Q4. Find the distance between the point $A(4, 2, -1)$ and the plane P whose equation is $3x - 4y + z + 6 = 0$ in Euclidian space \mathbb{R}^3 . / **4 marks**

Q5. The following are percentages of milk fats from 20 samples collected from the milk brought by famers to a milk-collecting centre

4.12 4.04 3.96 3.95 3.98 3.94 3.98 4.12 3.98 3.96

3.95 4.02 3.95 4.02 4.04 3.98 4.02 4.02 3.95 4.02

Calculate the standard deviation of the data. / **4 marks**

Q6. Solve in \mathbb{R} set the equation: $2 \ln(x + 1) = \ln(1 - x)$ / **3 marks**

Q7. Show that the three points $A(2, -1, 3)$, $B(4, 3, 5)$, and $C(6, 7, 7)$ are collinear. / **4 marks**

Q8. Calculate: $\int \frac{\tan x}{\sec x + \cos x} dx$ / **5marks**

Q9. Solve for x : $4^{5-9x} = \frac{1}{8^{x-2}}$ / **3 marks**

Q10. Two cars start out at the same point. One car start out driving north at 25 km/h . Two hours later, the second car starts driving east at 20 km/h . How long after the first car starts travelling does it take the two cars to be 300 km apart? / **4 marks**

Q11. Given the matrix $A = \begin{pmatrix} 3 & 1 & -3 \\ 1 & 2a & 1 \\ 0 & 2 & a \end{pmatrix}$

find the possible values of a such that matrix A is singular. / **3marks**

Q12. Evaluate the limit: $\lim_{x \rightarrow -4} \frac{\ln(x+5)}{x+4}$ / **3 marks**

Q13. Solve: $z^2 - 2(\cos \beta)z + 1 = 0$ / **2marks**

Q14. a) Determine whether the series $(U_n)_{n \in \mathbb{N}}$ given by $U_n = \frac{2n+6}{8}$ is arithmetic or geometric. / **2 marks**

b) Calculate $\sum_{n=1}^{20} U_n$ / 2marks

Q15. Utilising Moivre's theorem, express $\sin 5x$ as a polynomial in $\sin x$. / 3mark

Q16. A) The population(p) of Enzymes in a culture solution changes according to the equation:

$$\frac{dp}{dt} = \frac{3000}{1+0.25t}, \text{ where } t \text{ is the time in hours. The initial population when } t = 0 \text{ second is } 1000.$$

- i) Find the expression for the population (p). / 5marks
- ii) Find the number of enzymes after $t = 3$ hours / 5marks

B) Suppose that the profit p obtained in selling x units of certain item each week is given by:

$$p = 50\sqrt{x} - 0.5x - 500 \quad \text{where } 0 \leq x \leq 8000. \text{ Find the rate of change of } p \text{ with respect to } x \text{ when } x = 1600. / 5marks$$

Q17. The function f is defined by $f(x) = \frac{\sqrt{1-x^2}}{x}$

- a) Determine the domain of definition of the function f and the limits of the boundaries of the domain. / 4marks
- b) Write the equations of asymptotes if available. / 1mark
- c) Determine the first derivative and the second derivative of the function f . / 5marks
- d) Determine the variation of the function f , the point of inflexion and the nature of the curve representing the function f . / 3marks
- e) Sketch the graph of the curve of the function f . / 2marks

Q18. a) The events A, B and C in the same sample space are such that A and C are mutually exclusive events, while A and B are independent events. Given that $P(A) = \frac{2}{3}, P(C) = \frac{1}{5}, P(A \cup B) = \frac{4}{5}$ and $P(B \cup C) = \frac{13}{25}$. Find: $P(A \cup C), P(B), P(A \cap B)$ / 6marks

b) Prove that $\sqrt{2 + \sqrt{2 + \sqrt{2 + 2\cos 8x}}} = 2\cos x$ / 4marks

c) Solve the equation $(x + \sqrt{x})^4 - (x + \sqrt{x})^2 = 159600$ / 5marks

Q19. a) Consider $F = \{(x, 0, z), x, z \in \mathbb{R}\}$ and $G = \{(x, y, 0), x, y \in \mathbb{R}\}$

Find: i) Dim F ii) Dim G iii) Dim $(F+G)$ / 6marks

b) Verify Grassmann's formula. / 4marks

c) Consider $F = \{(x, 0, z), x, z \in \mathbb{R}\}$ and $G = \{(x, y, 0), x, y \in \mathbb{R}\}$. Find $F \cap G$ / 5marks

Q20. a) Solve the following system of equations using the Gauss Jordan elimination method.

$$\begin{cases} 2x + y - 3z = 2 \\ x - 2y + z = 3 \\ 3x - y - 3z = 3 \end{cases} \quad / 8marks \quad \text{b) Solve: } y'' - y' - 2y = \sin 2x \quad / 7marks$$

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