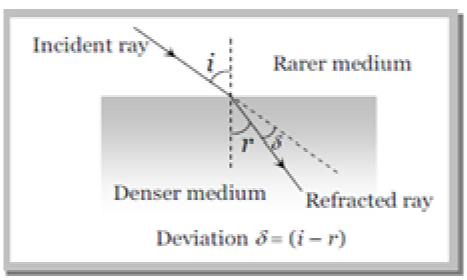
TOPIC AREA: LIGHT

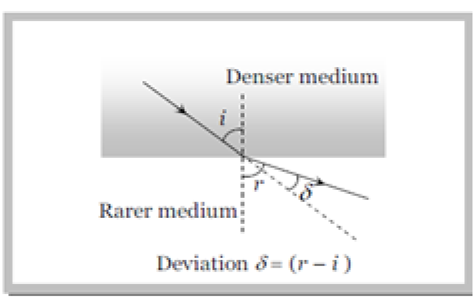
Unit 1: Thin lens

I. **Introduction to the refraction of light**

Refraction of light

Refraction of light is phenomena where the direction of light is changed when it crosses the boundary between two materials of different optical densities. It due to the change in the velocity of light as it passes from one medium into another.





Note: *If the light passes through less dense to more dense medium it bends towards normal.
*If the light passes through high dense to less dense medium it bends away the normal.

Snell's law

The ratio of sine of the angle of incidence ***(i)*** to the angle of refraction ***(r)*** is a constant called refractive index.

$$\frac{\sin i}{\sin r} = \frac{n_2}{n_1}$$

For two media, Snell's law can be written as

$$\frac{\sin i}{\sin r} = n = \frac{n_2}{n_1}$$

The Laws of Refraction

The incident ray, the refracted ray and the normal at the point of entry are all in the same plane.

The ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant for a particular wavelength (Snell's Law).

$$\frac{\sin i}{\sin r} = n \quad where \;\; n \; is \; the \; absolute \; value \;\; n = \frac{n_2}{n_1}$$

Or $n_1 \sin i = n_2 \sin r$

Example: A light ray, with an angle of incidence of 350, passes from water to air. Find the angle of refraction using Snell's Law. Discuss the meaning of your answer.(refractive index of water is 1.33 , for air it is 1).

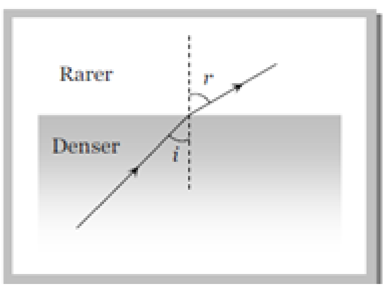
Soln

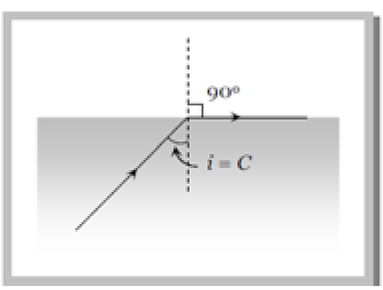
According to Snell's Law:

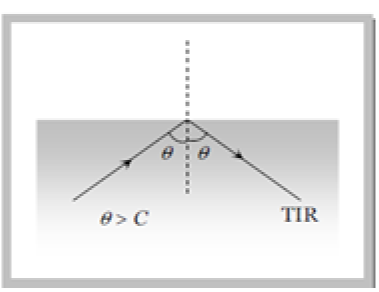
$$\begin{aligned} n_1 \sin\theta_1 &= n_2 \sin\theta_2 \\ 1{,}33 \sin 35^\circ &= 1 \sin\theta_2 \\ \sin\theta_2 &= 0{,}763 \\ \theta_2 &= 49{,}7^\circ. \end{aligned}$$

Total Internal reflection. When a ray of light goes from denser to rarer medium it bends away from the normal and as the angle of incidence in denser medium increases, the angle of refraction in rarer medium also increases and at a certain angle, angle of refraction becomes 90^0, this angle of incidence is called **critical angle** (C).

When Angle of incidence exceeds the critical angle than light ray comes back in to the same medium after reflection from interface.
This phenomenon is called **Total internal reflection**







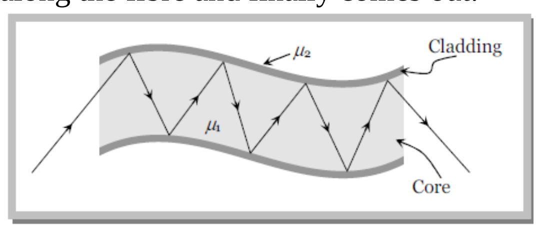
The refractive index is μ given by

$$\frac{n_2}{n_1} = \frac{1}{\sin C}$$

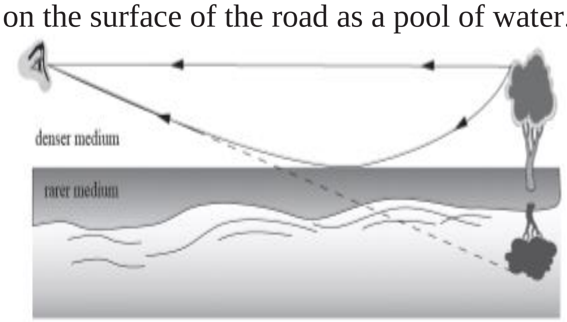
Examples of total internal reflection (TIR)

1. **Optical fibre**

Optical fibres consist of many long high quality composite glass/quartz fibres. Each fibre consists of a core and cladding. The refractive index of the material of the core n_1 is higher than that of the cladding (n_2).
When the light is incident on one end of the fibre at a small angle, the light passes inside, undergoes repeated total internal reflections along the fibre and finally comes out.

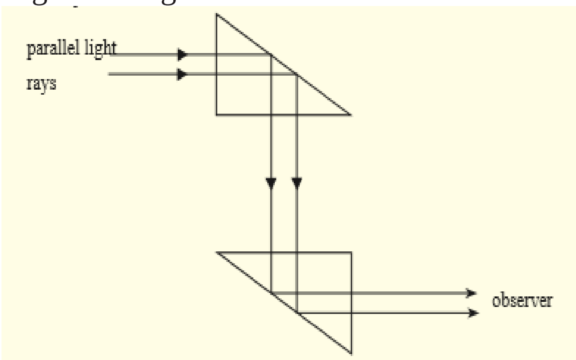


2.Mirrages
Mirrages are formed because air above the ground is hotter(less denser) than the air higher up, then the light from the sky and clouds is refracted gradually towards the horizontal after passing through the different layers of air decreasing density. As the the light hits the ground at an angle greater than the critical angle and total internal reflection occur, Hence light enters our eyes as if it is on the surface of the road as a pool of water.



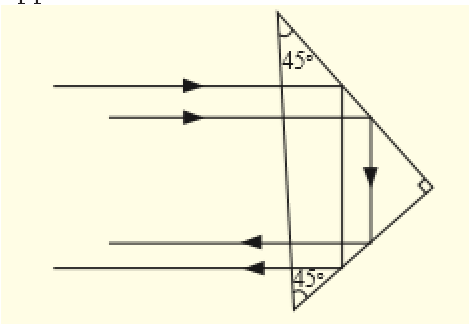
3. Periscopes
Periscopes are made by arranging two isosceles right angled triangle glass prism, if

the light enters one of the short faces at right angle, it is totally reflected as it strikes it at an angle of 45^0 which is greater than the critical angle of the glasss.



4. Prism Binoculars

In binoculars light is turned at an angle of 180^0 and made to travel (totally reflected) in opposite direction.



Application exercises

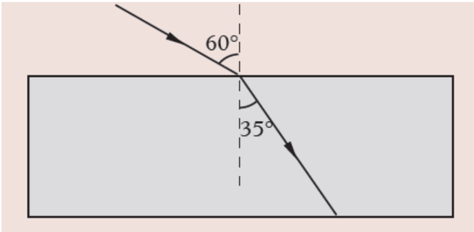
1.(a) Define critical angle.

(b) Write down the relationship between the critical angle and the refractive index of a medium.

2.State the two conditions under which total internal reflection occurs.

3.Calculate the value of critical angle for a liquid-air interface, if the refractive index of the liquid is 1.40.

4.In below calculate the refractive index of glass.



5.The velocity of light in glass is 2.0 × 10^8 m/s. Calculate (a) the refractive index of glass and (b) the angle of refraction in the glass for a ray of light passing from air to glass at an angle of incidence of 40°.

6.The angle of incidence for a ray of light passing from air to water is 30° and the angle of refraction is 22°. Calculate the refractive index of water.

7.Calculate the critical angle for glass-air interface, if the refractive index of glass is 1.50.

8.Calculate the critical angle at the water-air interface if the refractive index of water is 1.33

9.Calculate the refractive index of diamond, if the critical angle for the diamond is 24°.

Thin lenses

A lens is a transparent medium (usually glass) bounded by one or two curved surfaces.

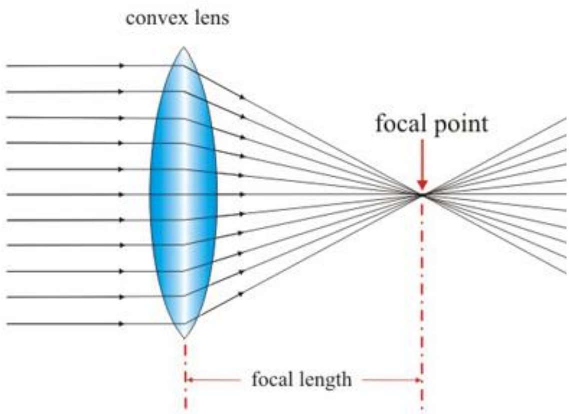
Types of lenses

There are two types of lenses; a convex lens also called a converging lens and a concave lens also known as a diverging lens.

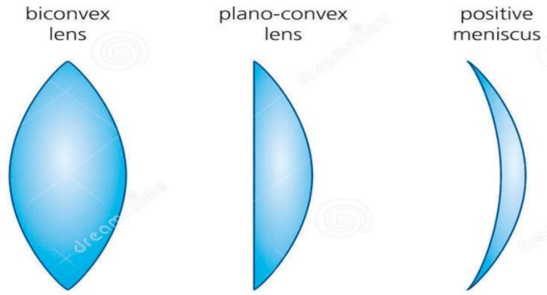
a) A convex lens

A convex lens is the one which is thicker at the center than at the edges.

Rays of light that pass through the lens are brought closer together (they converge). A convex lens is a **converging lens**.



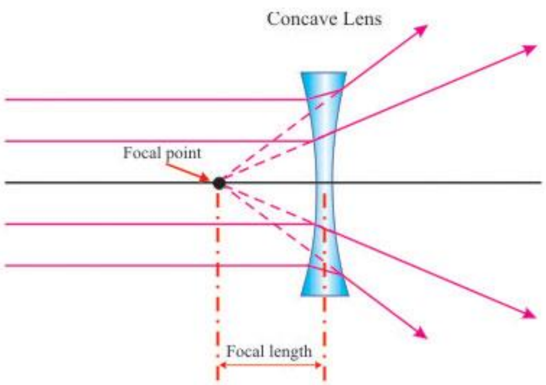
The figure below shows three types of convex lenses.



b) Concave lens

A concave lens is the one which is thinner at the center than at the edges.

Rays of light that pass through the lens are spread out (they diverge). A concave lens is a **diverging lens**.



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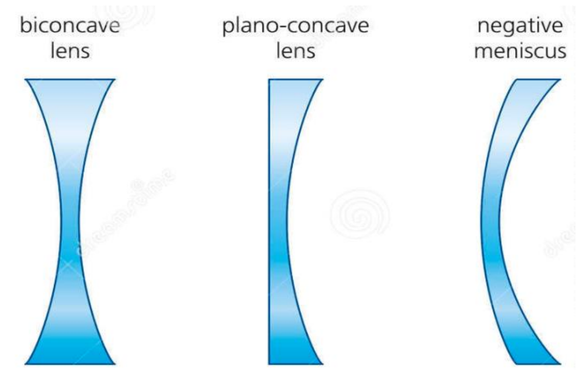
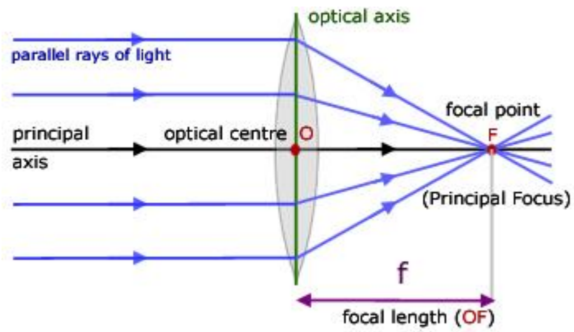
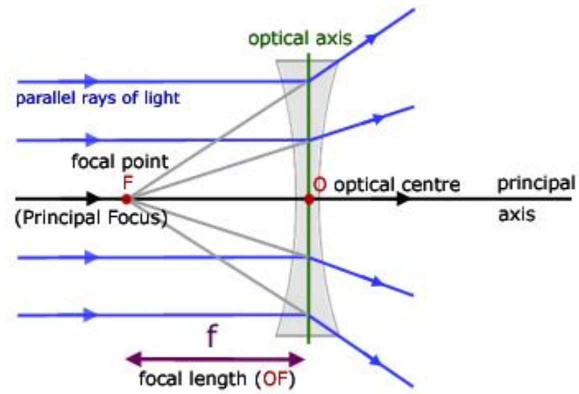


Term used for lens

The figures below shows Term used for lens





Principal axis: A line which passes through the center of the lens, perpendicular to the lens surface.

Principal focus: is a point on the principal axis where rays of light parallel to the the principal axis converge.

Focal length is the horizontal distance between the principal focus and the optical centre of the lens.

Optical centre - an imaginary point inside a lens through which a light ray is able to travel without being deviated

Centre of curvature:is the centre of the sphere of which the lens surface is part.

Properties of images formed by lenses

There are two types of image formed by lens such as real and virtual image

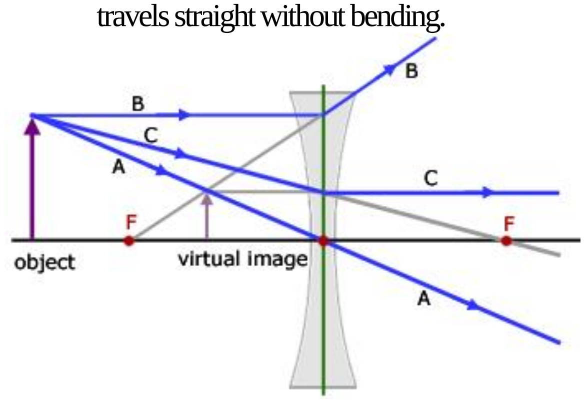
Real images - are produced from actual rays of light coming to a focus (e.g.: a film projected onto a screen)

Virtual images - are produced from where rays of light appear to be coming from (e.g.: a magnifying glass image)

Rules for a ray diagram

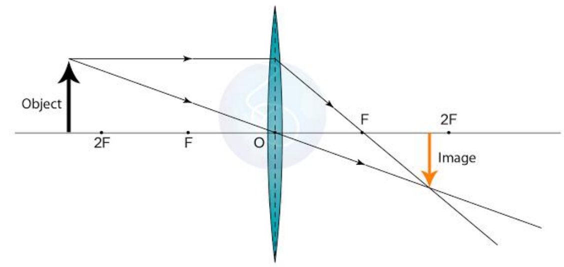
a) The ray parallel to the principal axis is refracted through the focus point, F.
b) A ray passing through the focus point is refracted parallel to the principal axis.

c) A ray passing through the optical centre travels straight without bending.



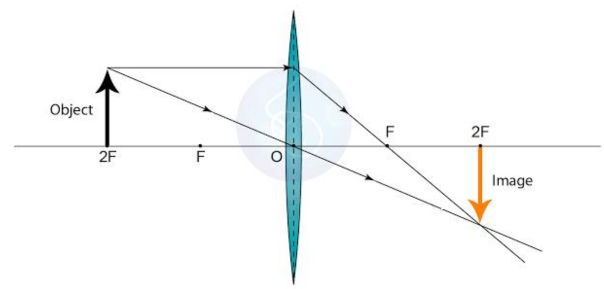
Graphical construction of images by diverging lens

a) Object beyond 2F



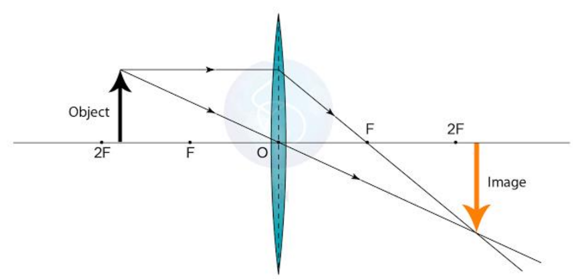
Characteristics of the Image: *Real, inverted, diminish.*

b) Object at 2F



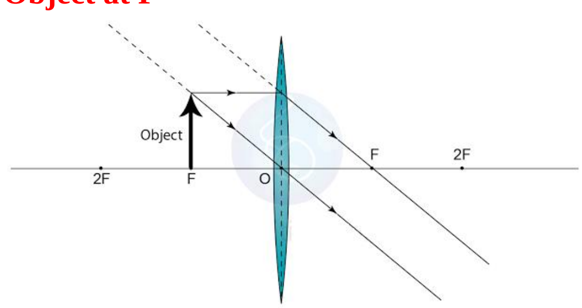
Characteristics of the Image: *Real, inverted, magnified.*

c) Object between F and 2F



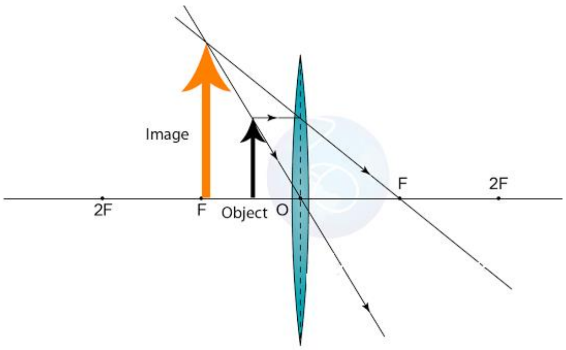
Characteristics of the Image: *Real, inverted, magnified*

Object at F



Characteristics of the Image: *infinity*

d) Object between F and optical centre



Characteristics of the Image: *Virtual, upright, magnified*

Graphical determination of focal length of a convex lens

At different values of ***u*** you obtain different values ***v*** are obtained.

From the lens formular

$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ *it follows that* $\frac{u+v}{uv} = \frac{1}{f}$ *thus* $f = \frac{uv}{u+v}$

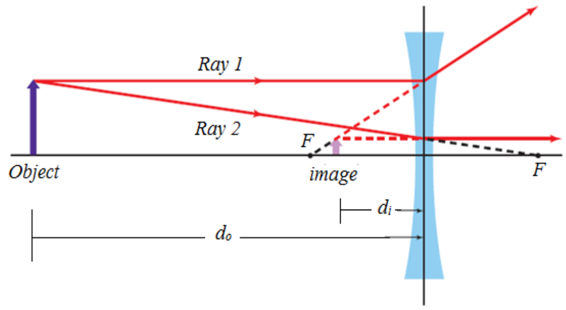
This implies that $uv = f(u+v)$. Where $u+v$ is the distance(d) between object and image. The expression is an equation of a line. Hence a graph of uv *against* $u+v$ is a straight line passing through the origin and its slope is **The focal length** $\boldsymbol{f}$ of the lens

Remark: Results must be recorded in a suitable table including values of uv & u + v.

u/cm	v/cm	u+v/cm	Uv/cm^2

Graphical construction of images by diverging lens

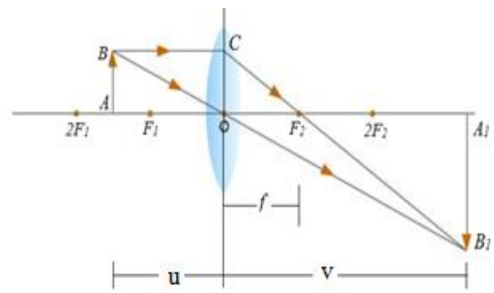
If the lens is biconcave or plano-concave, a beam of light travelling to the lens axis and passing through the lens will be diverged.



Concave lenses produce only virtual images that are upright and smaller compared to their objects.

The lenses formula

Convex lens



Let *AB* represent an object placed at right angles to the principal axis at a distance greater than the focal length *f* of the convex lens. The image A_1B_1 is formed beyond $2F_2$ and is real and inverted.

Triangles OAB and OA_1B_1 are similar

$$\frac{A_1B_1}{AB} = \frac{OA_1}{OA} \qquad (1.2.11)$$

Similarly triangles OCF_2 and $F_2A_1B_1$ are similar

$$\frac{A_1B_1}{OC} = \frac{F_2A_1}{OF_2}$$

But we know that $OC = AB$

The above equation can be written as

$$\frac{A_1B_1}{OC} = \frac{A_1B_1}{AB} = \frac{F_2A_1}{OF_2}$$

$$\frac{A_1B_1}{OC} = \frac{F_2A_1}{OF_2} \qquad (1.2.12)$$

From equation (1.2.11) and (1.2.12), we get

$$\frac{OA_1}{OA} = \frac{F_2A_1}{OF_2} = \frac{OA_1 - OF_2}{OF_2}$$

$$\frac{v}{u} = \frac{v-f}{f}$$

$$vf = uv - vf$$

Dividing equation above throughout by uvf we get

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \qquad (1.2.14)$$

Worked exercises

A 50 mm tall object is placed 12 cm from a converging lens of focal length 20 cm. What are the nature, size, and location of the image?

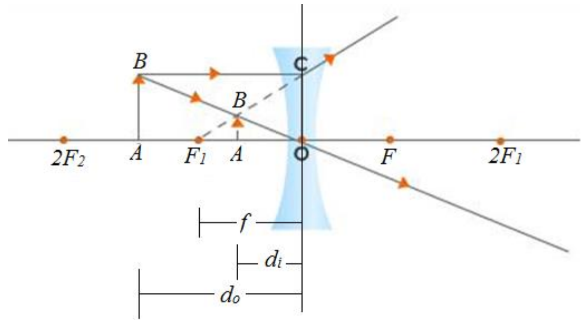
Solution

$$q = \frac{(12\text{ cm})(20\text{ cm})}{12\text{ cm - }20\text{ cm}}; \qquad q = \text{-30cm};$$

$$M = \frac{-q}{p} = \frac{y'}{y}; \quad y' = \frac{-qy'}{p} = \frac{-(-30\text{ cm})(50\text{ mm})}{12\text{ cm}}$$

q = -30.0 cm, y' = +125 mm, *virtual, erect, and enlarged*

2.4.3.2 Concave lens



The lens formula for concave lens is given in the same way as equation (1.2.15)

$$\frac{1}{f} = \frac{1}{d_i} - \frac{1}{d_o}$$

An object 6 cm high is held 4 cm from a diverging meniscus lens of focal length 24 cm. What are the nature and location of the image? Solution

$$q = \frac{pf}{p-f} = \frac{(4\text{ cm})(\text{-24 cm})}{4\text{ cm - (-24 cm)}};$$

q = -3.43 cm

q = -3.43 cm, y' = 5.14 cm, *virtual, erect, and diminished*

2.4.3.4 Magnification

The Cartesian magnification of lens is

$$m = \frac{h_i}{h_o} = \frac{A_1B_1}{AB} = \frac{OB_1}{OB} = -\frac{d_i}{d_0}$$

What is the magnification of a lens if the focal length is 40 cm and the object distance is 65 cm? Solution

$$q = \frac{(65\text{ cm})(40\text{ cm})}{65\text{ cm - }40\text{ cm}} = 104\text{ cm}; \qquad M = \frac{-q}{p} = \frac{-(104\text{ cm})}{(65\text{ cm})};$$

***M* = -1.60**

2.4.3.5 The power of lenses

Whenever a ray of light passes through a lens it bends except when it passes through the optical centre. The degree of convergence or divergence of a lens is expressed as power. A lens of short focal length deviates the rays more while a lens of large focal length deviates the rays less. Thus the power of a lens is defined as the reciprocal of its focal length.

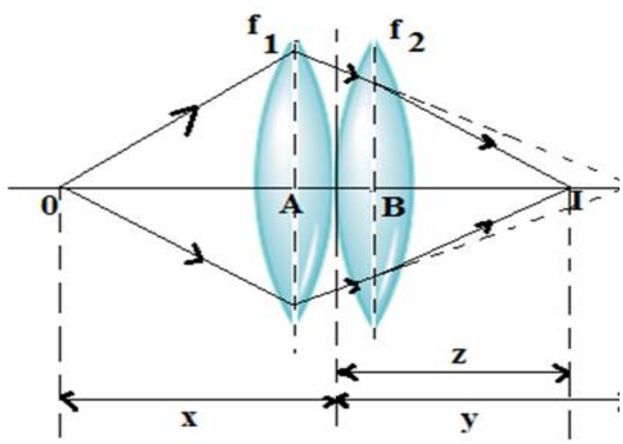
Power of a lens $P = \frac{1}{f}$

The unit of power is dioptre (D) $1D = 1m^{-1}$
In case the lenses are combined
$P = P_1 + P_2 + ...$

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + ... \qquad (1.2.18)$$

Combination of lens

Combinations of lenses in contact are used in many optical instruments to improve their performance . In figure below, A and B are two thin lenses in contact of focal lengths f_1 and f_2. Paraxial ray from point object O on the principle axis are refracted through A and would, in absence of B, give a real image of O at I'



For the lens A: u=+X and v=+y. From the thin lens formula equation we have:

$$\frac{1}{f_1} = \frac{1}{+x} + \frac{1}{+y} \quad (1)$$

For the lens B : u=-y(since I' is an imaginary object with respect to lens B) and v =+Z (image distance of I')

$$\frac{1}{f_1} = \frac{1}{-y} + \frac{1}{+z} \quad (2)$$

Adding we get: $\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{x} + \frac{1}{y} - \frac{1}{y} + \frac{1}{z}$ (3)

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{x} + \frac{1}{z} \qquad (4)$$

Considering the combination, I is the real image formed by O by both lenses, therefore u=+x and v=+z and

$$\frac{1}{f} = \frac{1}{x} + \frac{1}{z} \quad (5)$$

f is the combined focal length of a single lens that would be exactly equivalent to the two in contact.

Using eq. (4) into (5), we get $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

and $f = \frac{f_1 f_2}{f_1 + f_2}$ (6)

Therefore, two thin lenses in contact with each other are equivalent to a single thin lens having a focal length given by Equation (6).

Application exercise

Two converging lenses, with the focal length f_1 = 10 cm and f_2 = 15 cm are placed 40 cm apart. An object is placed 60 cm in front of the first lens as show in second figure.
a) Find the position of the final image formed by the combination of the two lenses?
b) Find magnification of the final image formed by the combination of the two lenses?

Lens maker's equation

The focal length of a lens in air can be calculated from the lens maker's equation:

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right).$$

The above equation helps to determine the focal length of meniscus lenses given the refractive index of the material and the radii of curvature r_1 and r_2 for the spheres that make the lens.

Remark: -If the **convex surface** is in less denser medium the radius is **positive (+R)**

-If the concave surface is in less denser medium, the radius is **negative (-R)**

N.B – A concave surface **converges** the light while a convex surface **diverges** light.

Example: A lens has a convex surface of radius 20 cm and a concave surface of radius of 40 cm and is made of glass of refractive index 1.54. Compute the focal length of the lens, and state whether it is a converging lens or a diverging lens.

Soln!

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \; but\; R_1 = +20; R_2 = -40$$

$$\frac{1}{f} = (1.54-1)\left(\frac{1}{20} - \frac{1}{40}\right) = (0.54)\left(\frac{1}{40}\right) =$$

$$f = \frac{40}{0.54} = 74.07cm. \quad it\ is\ a\ converging\ lens$$

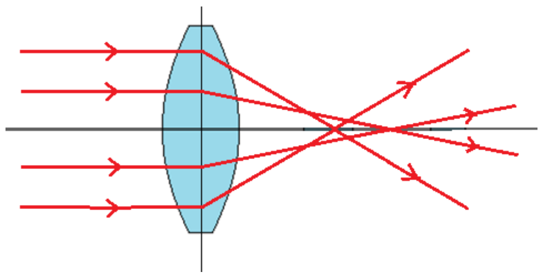
Defects of lenses: Lens aberrations

A lens aberration is an optical design image error. It's caused by the fact that in practice the lens medium can cause substantial deviation of light rays from the direction they are intended to travel in the lens' theoretical, ideal optical design.

There are several different types of aberration which can cause the image to be an imperfect replica of the object ;(**Spherical aberration, chromatic aberration, coma, field curvature, barrel, pincushion distortion, astigmatism**, etc).

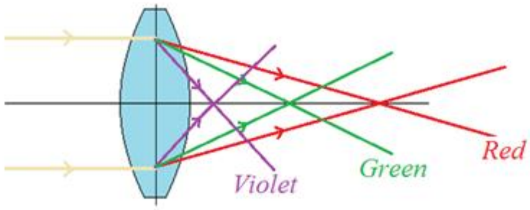
Spherical aberration

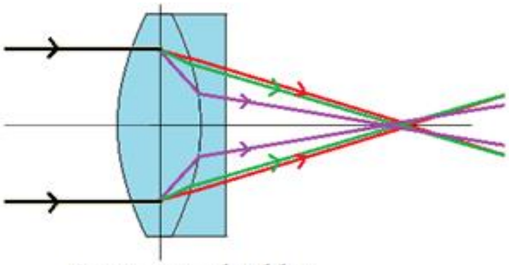
Spherical aberration comes into play when the marginal and Para-axial rays are unable to focus at the same point on the principal axis resulting in a blurring of the image.



2.4.4.2 Chromatic aberration

Chromatic aberration is caused by the dispersion of the lens material. Since, from the lens formulae, *f* is dependent upon *n*, it follows that different colours of light will be focused to different positions. Chromatic aberration can be minimized by using **an achromatic doublet (or achromatic)** in which two materials with differing dispersion are bonded together to form a single lens.

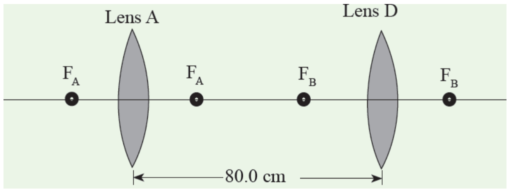




Acromatic doublet

Application exercises

1. An object AB of 1cm is placed at 8cm from a converging lens of focal length 12cm. Find its image (Position, nature and the size).
2. A thin glass lens n = 1.5 has a focal length +10cm in air. Compute its focal length in water n = 1.33.
3. A prism which has a refracting angle equals 60o and refractive index 1.5 receives a ray at an angle of incidence 45o; calculate the angle of emergence and the deviation of the ray.
4. An object of 2cm is placed at 50cm from a diverging lens of focal length 10cm. Determine its image.
5. An object located 32.0 cm in front of a lens forms an image on a screen 8.00 cm behind the lens. (a) Find the focal length of the lens. (b) Determine the magnification. (c) Is the lens converging or diverging?
6. Two converging lenses A and B, with focal lengths f_A=20cm and f_B = 25cm, are placed 80cm apart, as shown in the figure below. An object is placed 60cm in front of the first lens as shown in figure Determine (a) the position, and (b) the magnification, of the final image formed by the combination of the two lenses.



Refraction through glass prism

Terms associated with refraction of light passing through a prism

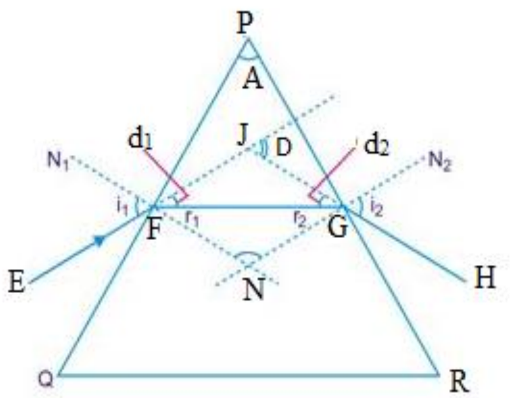
In optics, a prism is a transparent medium bounded by the three plane faces. Out of the three faces, one is grounded and the other two are polished. The polished faces are called **refracting faces**. The angle between the refracting faces is called **angle of prism**, or the refracting angle. The third face is called **base of the prism.**

Refraction through prisms

In optics, a prism is a transparent optical element with polished surfaces that refract light. The angle between surfaces is known as "**refracting angle"** or "**angle of prism".** The traditional geometrical shape is that of a **triangular** prism with a triangular base and rectangular sides.

Deviation of light by prism

A prism deviates light on both faces. These deviations do not cancel thus the total deviation of a ray due to refraction at both faces of the prism is the sum of deviation of a ray due to refraction at the first surface and its deviation at the second face.



Let d_1 and d_2 be the angles of deviation at the first and the second faces of the prism respectively.

Total deviation $D = d_1 + d_2$;

Angle JFG $(d_1) = i_1 - r_1$ and

Angle JGF $(d_2) = i_2 - r_2$

Angle FJG $= 180 - (i_1 - r_1 + i_2 - r_2)$

The angle of deviation

$$D = 180 - [180 + i_1 - r_1 + i_2 - r_2]$$

$$= (i_1 + i_2) - (r_1 + r_2)$$

Angle FNG $= 180 - (r_1 + r_2)$

In figure PFNGP, we have:

$$A + 90 + 180 - (r_1 + r_2) + 90 = 360$$

$$r_1 + r_2 = A$$

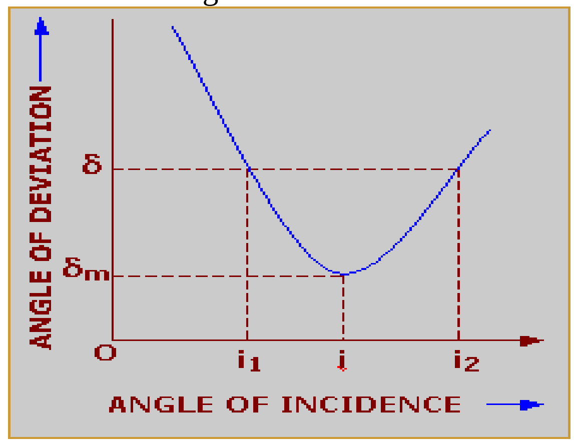
Therefore

$$D = (i_1 + i_2) - A$$

Angle of minimum deviation and determination of refractive index of the prism

The angle of deviation of a ray of light in passing through a prism not only depends upon its material but also upon the angle of incidence.

The above figure shows the nature of variation of the angle of deviation with the angle of incidence. It is clear that an angle of deviation has the minimum value $D_{\min}$ for only one value of the angle of incidence. The minimum value of the angle of deviation when a ray of light passes through a prism is called the angle of minimum deviation



Notes: Minimum deviation occurs when the angle of emergence of the ray from the second face equals to the angle of incidence of the ray on the first face.

At the minimum deviation, $i_1 = i_2 = i$ and $r_1 = r_2 = r$

Therefore

$$D_{\min} = 2i - A \text{ and } 2r = A$$

Whereby $i = \dfrac{D_{\min} + A}{2}$

$$A = \frac{r}{2}$$

The refractive index of the material of the prism, n, is given by:

$$n = \frac{\sin i}{\sin r}$$

This implies that:

$$n = \frac{\sin\left(\dfrac{D_{\min} + A}{2}\right)}{\sin\left(\dfrac{r}{2}\right)}$$

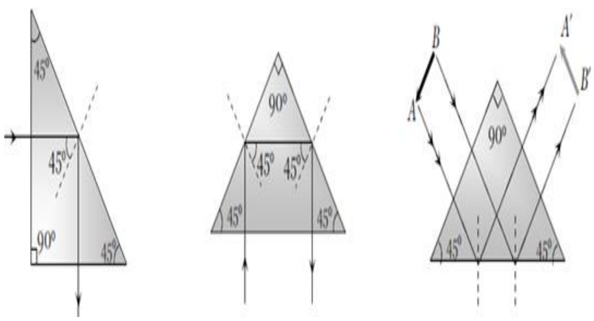
For small angles A and $D_{\min}$

$$\sin\frac{D_{\min} + A}{2} \approx \frac{D_{\min} + A}{2} \qquad \sin\frac{A}{2} \approx \frac{A}{2}$$

$$D_{\min} = A(n-1)$$

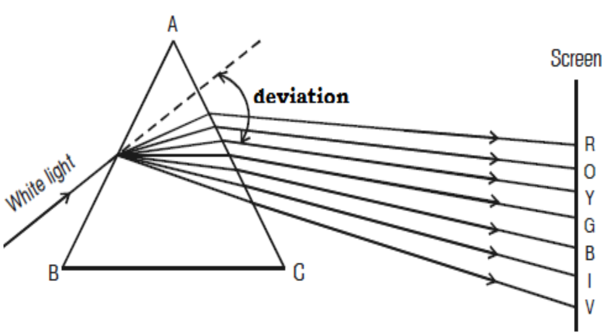
Total internal reflection by prism

A right angled isosceles prism , which is used in periscopes or binoculars.
It is used to deviate light rays through $90^0$and 180^0 and also to erect the image.



1.1 Dispersion of light by a prism

Dispersion is the splitting of white light into its constituent colours. This band of colours of light is called its **spectrum**.



It was in the year 1686 that Sir Isaac Newton did his well known experiments on refraction of white light by a glass prism. He observed that a beam of white light incident on a prism splits into its constituent colors to form “a **visible spectrum (bundle of colours)**.
In the visible region of spectrum, the spectral lines are seen in the order from violet to red. The colours are given by the word VIBGYOR (**Violet, Indigo, Blue, Green, Yellow, Orange and Red)**

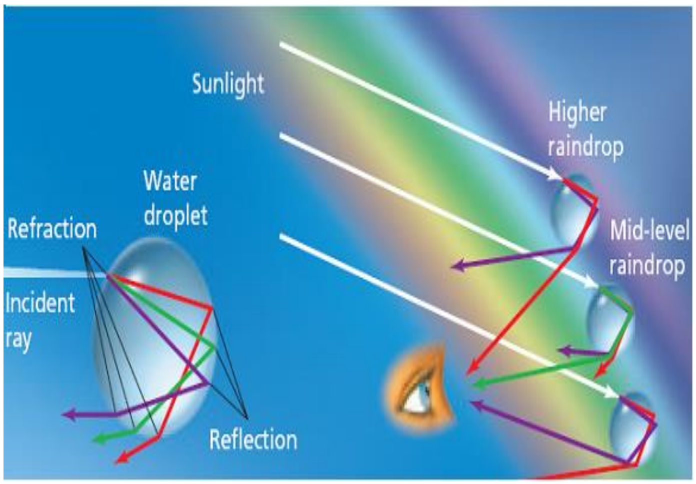
Remarks:

***Violet** color suffers the maximum deviation and **red the least**. ($D_R < D_V$)
* Speed red is greater than for violet ($V_R > V_v$
* Refractive index for red is less than that for violet($n_{R} < n_V$)

1.10. Example of dispersion

1.11. Rainbow

A rainbow is a spectrum formed when sunlight is dispersed by water droplets in the atmosphere. Sunlight that falls on a water droplet is refracted. Because of dispersion, each color is refracted at a slightly different angle. At the back surface of the droplet, some of the light undergoes internal reflection. On the way out of the droplet, the light once again is refracted and dispersed.

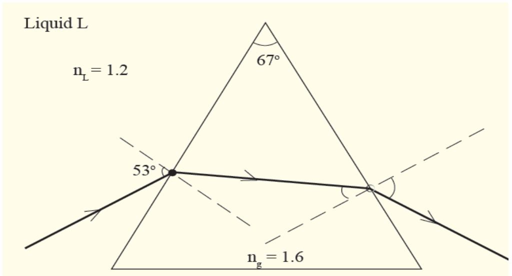


NOTE: *Rainbow as a phenomena is formed due **Total internal reflection** and **Dispersion**.

*While dispersion caused by the variation of refractive index of each colour.

Exercises

1. A glass prism of refracting angle 60^o has a refractive index of 1.5.Calculate the angle of minimum deviation for a parallel beam of light passing through it



2. A glass prism of refracting angle 72^0 and index of refraction 1.66 is immersed in a liquid of refractive index 1.33. What is the angle of minimum deviation for a parallel beam of light passing through the prism?

3. A glass prism of refracting angle 60^0 has a refractive index of 1.5. Calculate the angle of minimum deviation for a parallel beam of light passing through it.

4. A mono chromatic light is incident on one refracting surface of a prism of refracting angle 60o, made of glass of refractive index 1.50. Calculate the least angle of incidence for the ray to emerge through the second refracting surface.

5. A ray of light incident from air to a prism of refracting angle 60^0 grazes the boundary on the second face of the prism. Find the angle of incidence of the ray on the first face. (Take $n_g = 1.52$).

6. A ray passes through an equilateral prism such that the angle of incidence is equal to the angle of emergence and the later is equal to 3/4 of the angle of prism. Find the angle of deviation.

7. A thin prism of refractive index 1.5 deviates a ray by a minimum angle of 5^o. When it is kept immersed in oil of refractive index 1.25, what is the angle of minimum deviation?

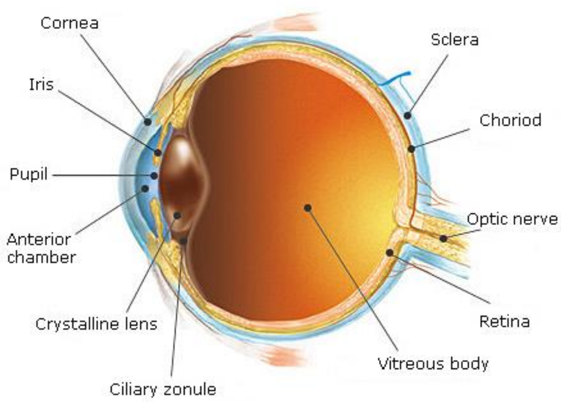
Unit 2: Optical instruments

.1 Simple Optical Instrument

2.1.1 The human eye

The eye is a biological instrument used to see objects at different distances. It uses a convex lens system to form a small, inverted, real image of an object in front of it.

Structure of the eye



The cornea: it is made out of a fairly dense, jelly like material which provides protection for the eye, and seals in the aqueous humour.

The aqueous humour: this is a watery liquid that help to keep the cornea in a rounded shaped, similar to that of a lens.

The iris: The iris is the coloured part of the eye that controls the amount of the light entering the eye.

The lens: this is used to focus an image on retina.

The ciliary muscles: these control the thickness of the lens during focusing.

The retina: The retina may be described as the "screen" on which an image is formed by light that has passed into the eye via the cornea, aqueous humour, pupil, lens, then the hyaloids and finally the vitreous humour before reaching the retina.

The retina contains photosensitive elements (called **rods** and **cones**) that convert the light they detect into nerve impulses that are then sent onto the brain along the optic nerve

The vitreous humour: this is a jerry like substance that helps the eye to keep its round shape. It is very close in optical density to the lens material.

The optical nerve: this is the nerve that transmits images received by the retina to the brain for interpretation.

Accommodation of the eye

Accommodation of the eye is the ability of the eye to see near and distant objects. The eye is capable of focusing objects at different distances by automatic adjustment of the thickness of the eye lens which is done by the ciliary muscles.

Near point and far point of the eye

The near point of the eye *is the nearest point that can be focused by the unaided eye.* It is a closest distance that a normal human eye can observe clearly without any strain to the eye. It is called the least distance of distinct vision. The near point of a normal eye is 25cm.

Far point of the eye

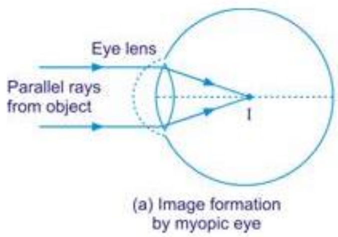
The far point is the farthest point that can be focused by the eye. The far point of the eye is *infinity*.

Defects of Vision and their Correction

There are four types of defect of the Eye: Myopia, Hypermetropia, Presbyopia andAstigmatism. Below are given the nature of the defect, its causes and corrective measures

Myopia

Nearsightedness, also called myopia is common name for impaired vision in which a person sees near objects clearly while distant objects appear blurred. In such a defective eye, the image of a distant object is formed in front of the retina and not at the retina itself.

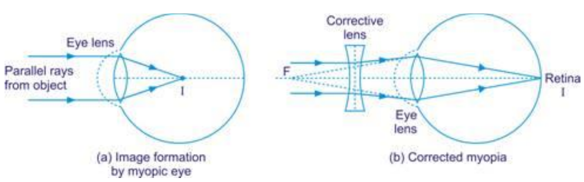


Causes:

This defect arises because the power of the eye is too great due to the decrease in focal length of the crystalline lens. This may arise due to either

i. excessive curvature of the cornea, or
ii. Elongation of the eyeball.

Correction:

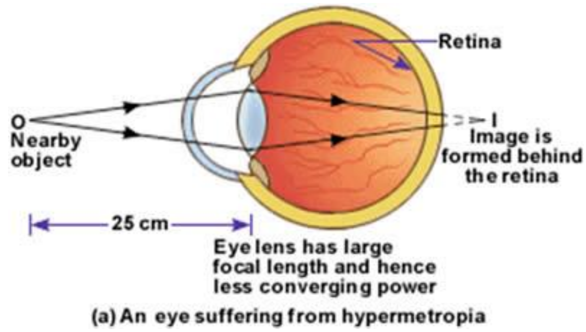


This *defect* can be *corrected* by using a *concave (diverging*) lens. A concave lens of appropriate power or focal length is able tobring the image of the object back on the retina itself.

Hypermetropia

Farsightedness, also called hypermetropia, common name for a defect in vision in which a person sees near objects with blurred vision, while distant objects appear in sharp focus.

In this case, the image is formed behind the retina.



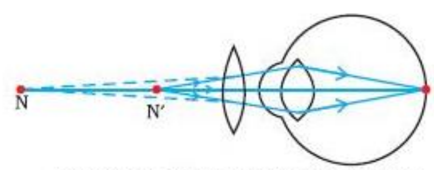
Causes:

This defect arises because either

i. the focal length of the eye-lens is too great, or
ii. The eyeball becomes too short, so that light rays from the nearby object, say at point N, cannot be brought to focus on the retina to give a distinct image.

Correction

This defect can be corrected by using a *convex* (*converging*) l*ens* of appropriate focal length.



Correction for Hypermetropic eye

When the object is at N', the eye exerts its maximum power of accommodation. Eyeglasses with converging lenses supply the additional focusing power required for forming the image on the retina.

Presbyopia:

This defect of vision usually happens is old age when ciliary muscles become weak and can no longer adjust the eye-lens.

The muscles become inflexible in this condition and cannot see nearby object clearly.

The near-point of an old person having presbyopia is much more than 25cm. presbyopia corrects by wearing spectacles having convex lens

N.B: Sometimes, a person may suffer from both *myopia* and *hypermetropia*. Such people often require *bi-focal lenses*. In the bi-focal lens, the upper portion of the bi-focal lens is a concave lens, used for distant vision. The lower part of the bi-focal lens is a convex lens, used for reading purposes.

Astigmatism

Astigmatism, a defect in the outer curvature on the surface of the eye that causes distorted vision. In *astigmatism*, a person cannot simultaneously focus on both horizontal and vertical lines.

***Causes*:**

This defect is usually due to the cornea that is not perfectly spherical. Consequently, it has different curvatures in different directions in vertical and horizontal planes. This results in objects in one direction being well-focused, while those in a perpendicular direction not well focused.

Correction

This defect can be corrected by using eyeglasses with *cylindrical lenses* oriented to compensate for the irregularities in the cornea

Exercises

1. A nearsighted person has a far point that is 323 cm from her eye. If the lens in a pair of glasses is 2.00 cm in front of this person's eye, what focal length must it have to allow her to focus on distant objects?
2. A farsighted person wears glasses to enable him to read a book held at a distance of 25.0 cm from his eyes, even though his near-point distance is 57.0 cm. If his glasses are at a distance of 2.00 cm from his eyes, find the focal length of his lenses to place the image of the book at the near point.
3. Find the focal length of a pair of contact lenses that will allow a person with a near point distance of 145 cm to read a newspaper held at 25.1 cm from his eyes.
4. A nearsighted person cannot see objects clearly beyond 25.0 cm (her far point). If she has no astigmatism and contact lenses are prescribed for her, what (a) power and (b) type of lens are required to correct her vision?
5. The near point of a person's eye is 60.0 cm. To see objects clearly at a distance of 25.0 cm, what should be the (a) focal length and (b) power of the appropriate corrective lens? (Neglect the distance from the lens to the eye.)

2.1.2 Simple microscope

A magnifying glass consists of a thin converging lens and it is used to view a very small objectsor a part of an organism which cannot be easily seen by a naked eye.

2.1 Formation of image by a magnifying glass

A magnifying glass forms a ***virtual, upright*** *and* ***magnified*** image of an object placed between the lens and the principle focus

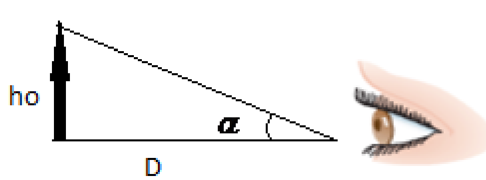
Magnifying power (angular magnification) of a simple microscope

The magnifying power- is defined as the as the ratio of the angle subtended at the eye by the image (when using an instrument) to the angle subtended at the eye by the object at the near point (with naked eye).

A.Magnifying power (angular magnification) of a simple microscope in normal adjustment

A simple microscope is said to be in normal adjustment when the final image is formed at near point.

*Consider an object of height ho placed at a distance D

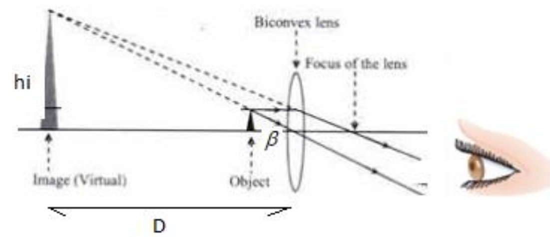


$\tan\alpha = \frac{h_o}{D}$; For small angle $\tan\alpha \approx \alpha$

Then the equation (1) becomes

$$\alpha = \frac{h_o}{D}$$

❖ Consider also, a simple microscope in normal adjustment



$\tan\beta = \frac{h_i}{D}$;For small angle $\tan\beta \approx \beta$

Then, the equation (2) becomes

$$\alpha = \frac{h_i}{D}$$

From the definition of magnifying power

$$M = \frac{\beta}{\alpha} \quad (3)$$

Using (1) and (2) into (3), we have

$$M = \frac{hi/D}{ho/D} \text{ i.e } M = \frac{hi}{D}\times\frac{D}{ho}$$

But $\frac{hi}{ho}$ is linear magnification,m, ;

$$m = \frac{hi}{u} = \frac{v}{u} = \frac{D}{U}$$

From the lens formula

$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ or $v = -D$ (for virtual image), we obtain

$$\frac{1}{u}-\frac{1}{D}=\frac{1}{f}$$

Multiply D both sides, leads to,

$\frac{D}{u}+\frac{D}{v}=\frac{D}{f}$ or $\frac{1}{f}=\frac{D}{u}-1 \leftrightarrow \frac{D}{f}=\frac{D}{u}-1$

$\frac{D}{u}=\frac{D}{f}+1$, or $M = \frac{D}{u}$

Therefore, angular magnification $\boldsymbol{M = \frac{D}{f} + 1}$

$M_D = \left(1+\frac{D}{f}\right)_{max}$ Where$D = +25cm$

A simple microscope is made of a combination of two lenses in contact of powers +15D and +5D. Calculate the magnifying power of the microscope, if the image is formed at 0.25m, the least distance of distinctvision.

Solution : Powers of the two lenses are p1 = + 15 D

and p2 = + 5D

D = distance of distinct vision = 0.25 m

∴ Power of the combination, P = + 15 + (+5) = + 20 D

∴ Focal length of the lens, f = 1/(+20) = 0.05 m

Now, magnifying power of simple microscope,

M = (1+ D/f)

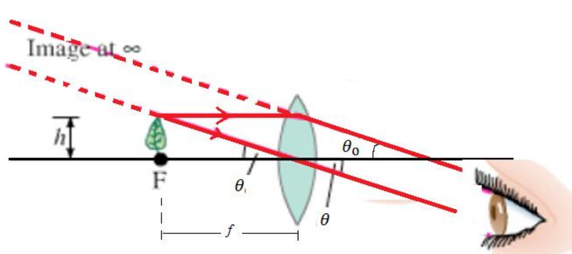
= 1 + 0.25/0.05

= 1 + 5 = 6 (Ans.)

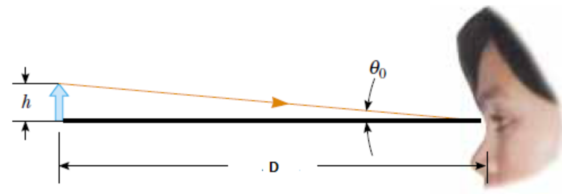
A.Magnifying power (angular magnification) of a simple microscope not in normal adjustment (not in normal use).

-The object has to be at the focal point of the lens.

-The eye is relaxed and the image is at infinity.



If the object, is viewed at near point by the unaided eye, (see fig. below)



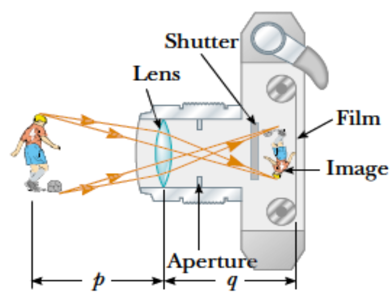
(a)

$\theta_0 \approx \frac{h}{D}$, and $\theta \approx \frac{h}{f}$ and the minimum magnification is $\boldsymbol{M_{min}} = \frac{\boldsymbol{\theta}}{\boldsymbol{\theta_0}} = \frac{\boldsymbol{D}}{\boldsymbol{f}}$

2.1.3 Lens camera

A photographic camera consists of

- A converging lens
- A light sensitive film at the other end
- A focusing device for adjusting the distance of the lens from the film an exposure arrangement which provides the correct exposure

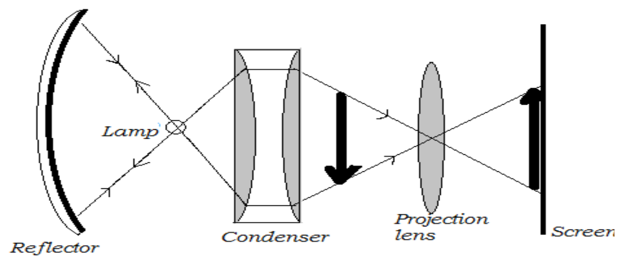


The shutter opens and closes quickly, thereby exposing the film to light for a short time to light entering the camera.

The object is placed in such a way that a real inverted image of the object is formed on the film.

2.1.4 The slide projector

A slide projector is a device used to throw on a screen a magnified image of a film or a transparent slide. It produces a magnified real image of an object.



It consists of an illumination system and a projection lens. The illumination system consists of a lamp, concave reflector and the condenser.

A lamp made in carbon electric arc or in quartz gives a small but very high intensity of light in order to make the image brighter. This lamp located at the center of curvature of a spherical mirror reflects back the light along their original path.

The condenser made by two plano-convex lenses collects light that is spread out towards the film (silde).

The light is then scattered b the film and focused by a convex projection lens on to the film.The projection lens is mounted in the sliding tube so that it is moved to and fro to focus a sharp image on the screen.

The linear magnification(or linear scale factor) of the projetor is given by the square root of area scale factor(or area magnification of image).

$$m = \sqrt{M_i}$$

Where m is a linear scale factor or linear magnification

M_i is the area scale factor or area magnification of image.

$$M_i = \frac{Area\ \ image}{Area\ of\ object}$$

$$M_i = \frac{A_i}{A_o}$$

Application exercises

Example:

A slide projector has a converging lens of focal length 20cm and is used to magnify the area of a slide, $5cm^2$ to an area of $0.8m^2$ on a screen.

Calculate the distance of the slide from the projector lens.

Solution

linear scale factor

$$m = \sqrt{M_i} = \frac{v}{u}$$

$$M_i = \frac{A_i}{A_o}$$

$$M_i = \frac{0.8}{0.0005} = 1600$$

$$m = \sqrt{1600} = \frac{v}{u}$$

Or $40 = \frac{v}{u}$ or $v = 40u$

Using lens formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

We have $\frac{1}{20} = \frac{1}{u} + \frac{1}{40u}$

And $u = 20.5cm$

2.2 Compound Optical Instrument

2.2.1 Compound microscope

The compound microscope consists of two optical components **i.e** Two convex lens separated by a distance L that is much greater than either eyepeice focal (f_e) or objective focal f_o (thus the term compound):

The objective lens system, which has a very short focal distance and is placed very close to the object; and the eyepiece system, which has a longer focal length, lower magnification;

The objective lens forms a **real**, **inverted image** and this image acts as an object for eyepieces lens

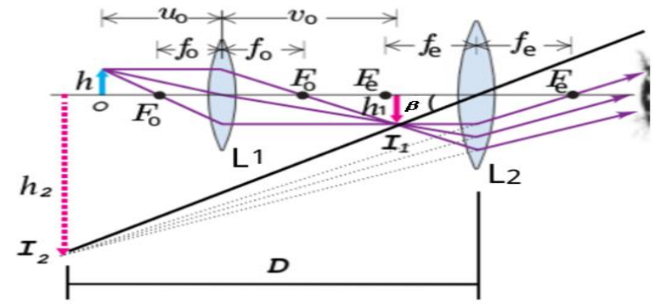
The eyepiece lens forms a **virtual** and **magnified** image

Magnifying power of compound microscope

Magnifying power of the microscope is defined as the ratio of the angle subtended by the image at the eye as seen through the microscope to the angle subtended by object at the unaided eye when both are placed at the least distance of distinct vision

a) *Compound microscope in normal adjustment*

A compound microscope is said to be in normal adjustment when the final image is formed at near point. The separation of the lenses is in such way that the intermediate is formed inside *fe*, so that the eyepiece acts as a magnifying glass.



The angular magnification, M of the microscope is given by

$$M = \frac{\beta}{\alpha}$$

$\beta =$ The angle subtended at the eye by the image

$\alpha =$ The angle subtended at the unaided eye by the object

- From the figure

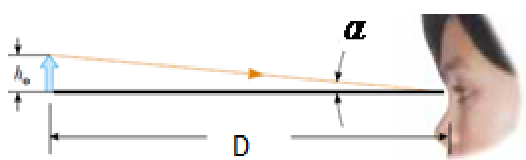
$$\tan\beta = \frac{hi}{D}$$

- For small angle $\tan\beta \approx \beta$

Therefore

$$\beta = \frac{hi}{D}$$

If the object were at the near point, it would subtended the small angle



$$\alpha = \frac{ho}{D}$$

From the definition of angular magnification

$$M = \frac{hi/D}{h_0/D}$$

$$M = \frac{hi}{h_o}$$

Multiply and dividing both side by $\frac{h}{h}$, we get

$$M = \frac{h}{h_o} \times \frac{h_i}{h},$$

Where $m_1 = \frac{h}{h_0}$: Linear magnification produced by objective lens, and

$m_2 = \frac{h_i}{h}$: Linear magnification produced by eyepiece lens.
Therefore $M = m_1 \times m_2$

u_0 Distance of object from objective
v_o = Distance image $A'B'$ formed by objective lens
u_e = Distance of $A'B'$ from eye lens,
v_e = Distance of final image from eye lens,
f_o = Focal length of objective, f_e = Focal length of eye lens.

From the objective lens, a real image is formed.

$$\frac{1}{u_o} + \frac{1}{v_o} = \frac{1}{f_o}$$

Multiply both side by $\mathbf{v_o}$

$$\frac{u_0}{u_o} + \frac{v_0}{v_o} = \frac{v_o}{f_o} \leftrightarrow \frac{v_0}{u_o} = \frac{v_o}{f_o} - 1$$

Therefore

$$m_0 = \frac{v_o}{f_o} - 1$$

From the eyepiece lens a virtual image is formed.

The eyepiece lens formula

$$\frac{1}{f_e} = \frac{1}{u_e} - \frac{1}{D}, \text{ or } D = -ve$$

$$\frac{D}{f_e} = \frac{D}{u_e} - \frac{D}{D} \leftrightarrow \frac{D}{f_e} = \frac{D}{u_e} - 1,$$

Therefore $\boldsymbol{m_2} = \frac{\boldsymbol{D}}{\boldsymbol{u_e}} = \frac{\boldsymbol{D}}{\boldsymbol{f_e}} + \mathbf{1}$

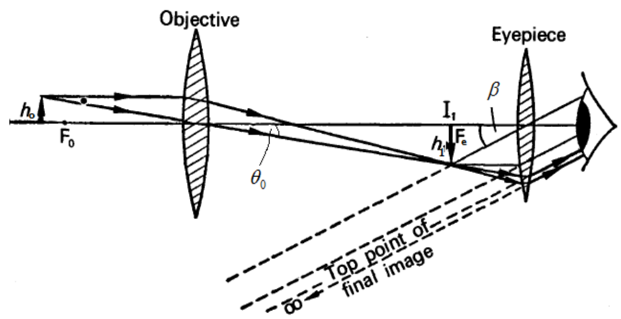
Total magnification: $M = m_1 \times m_2$

$M = \left(\frac{v_o}{f_o} - 1\right)\left(1 + \frac{D}{f_e}\right)$ at near point.

Angular magnification of compound micoscope when the final image is at infinity

The intermediate image is formed at the focal point of the eyepiece.

The final image is at infinity, $m_e = \frac{h_1}{f_e}$ since the eyepiece acts as a simple microscope which is not in normal use. The eye is at rest.



If the object were at the near point, it would subtended the small angle

$$\theta_0 = \frac{h_o}{D}$$

$$M_\infty = \frac{\beta}{\theta_0} = \frac{h_1}{f_e} \times \frac{D}{h_o} = \frac{h_1}{h_o} \times \frac{D}{f_e} = m_o \times \frac{D}{f_e}$$

$m_o = \left(\frac{v_o}{u_o} - 1\right)$ is the magnification of the objective.

Then $M_\infty = \left(\frac{v_o}{u_o} - 1\right)\frac{D}{f_e}$

Application exercises

A compound microscope has an eye piece of focal length 2.5cm and an objective of focal length 1.6cm. If the distance between the objective and eyepiece is 22.1cm, calculate the magnifying power produced when the final image is at infinity.

Solution

If the final image is at infinity, the objective forms an image at the focal point of the eyepiece.

Position of an image formed by objective=separation-focal length of eyepiece

$$v = 22.1 - 2.5 = 19.5cm$$

Magnifying power

$$M = \left(\frac{v_o}{f_o} - 1\right)\left(\frac{D}{f_e}\right)$$

$$M = \left(\frac{19.5}{1.6} - 1\right)\left(\frac{25}{2.5}\right) = 111.8$$

Test yourself

1. A microscope has its objective and eyepiece 18 cm apart. If $f_{obj} = 0.40$ cm and $f_{eye} = 5.0$ cm, where must a specimen be located to produce a final virtual image at infinity? What is the total magnification of this microscope?

2. A compound microscope has an objective with a power of 45 D and an eyepiece with a power of 80 D. The lenses are separated by 28 cm. Assuming that the final image is formed 25 cm from the eye, what is the magnifying power

3. A microscope has a magnifying power of 600, and an eyepiece of angular magnification of 15. The objective lens is 22 cm from the eyepiece. Without making any approximations, calculate (a) the focal length of the eyepiece, (b) the location of the object such that it is in focus for a normal relaxed eye, and (c) the focal length of the objective lens.

4. A microscope has an objective lens of focal length 5.00 mm. The objective forms an image 16.5 cm from the lens. The focal length of the eyepiece is 2.80 cm. Objective image is formed at the focal point of the eyepiece. What is the total angular magnification? The near point is 25.0 cm.

5. A microscope has a 13.0 x eyepiece and a 57.0 x objective lens 20.0 cm apart. Calculate the focal length of each lens. Where the object must be for a normal relaxed eye to see it in focus?

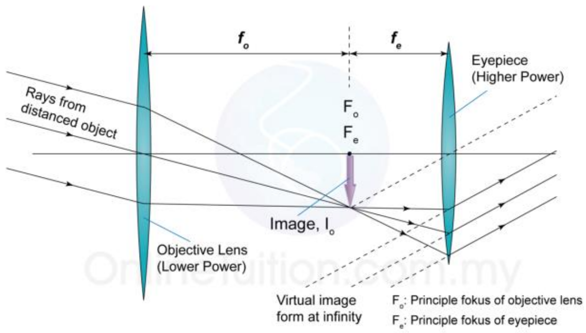
2.2.2 Refracting astronomical telescope

An astronomical telescope is an optical instrument which is used to see the magnified image of distant heavenly bodies like stars, planets, satellites and galaxies etc

It consists of two convex lenses called **objective** and **eyepiece**. The objective is of large focal length whereas the eyepiece is of short focal length *(fo > fe)*

. The distance between the two lenses can be adjusted by adjusting the tube which holds the lens.

Consider three rays **from the "top" point** of a very distant object. (The "bottom" point of the object is assumed to be **on the principal axis**.) See diagram below.

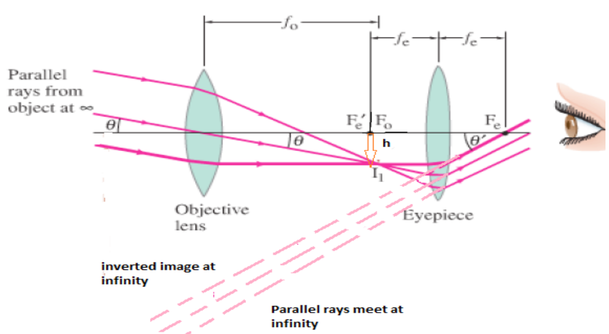


Magnifying power of an astronomical

Magnifying power of an astronomical telescope may be defined as the ratio of the angle subtended at the eye by the image to the angle subtended at the eye by the object

Magnifying power of refracting telescope when final image is at infinity (normal adjustment)

The object is at infinity, and therefore, the intermediate image is in the focal plane of the objective lens. The separation of the lenses is such that their focal planes coincide, and therefore, the eyepiece lens acting as magnifying glass, produces a final image which is at infinity. ***The eye is relaxed***.



For good approximation

$\tan\beta \approx \beta$, and $\tan\alpha \approx \alpha$
βis angle subtended at the eye by image
α *is* angle subtended at unaided eye by object
.
Since both the object and the final image are at infinity, the angles they subtend at the unaided eye are the same as those they subtend at the objective and at the eyepiece respectively. It follows that β and α are shown in figure above, from which

$\alpha = \frac{h}{fo}$, and $\beta = \frac{h}{fe}$

Therefore, since $M = \frac{\theta'}{\theta}$ then $M = \frac{h \div f_e}{h \div f_o}$

Or $\mathbf{M} = \frac{\mathbf{f_o}}{\mathbf{f_e}}$

Length between the two lenses $\boldsymbol{L = f_o + f_e}$

Application exercises

Magnification produced by astronomical telescope for normal adjustment is 20 and length of telescope is 1.05*m*. The magnification when image is formed at least distance of district vision (*D*=25*cm*)is

(*a*)6 (*b*)10 (*c*)14 (*d*)24

$M_1 = 20 = \frac{f_0}{f_e}$ ----(1)

Also (1) and (2) $f_0 + f_e = 1.05\ m$ ----(2)

Solving (1) and (2) we get,

$f_0 = 100\ cm$ and $f_e = 5\ cm$

Then $M_2 = f_0\left[\frac{1}{D} + \frac{1}{f_e}\right]$

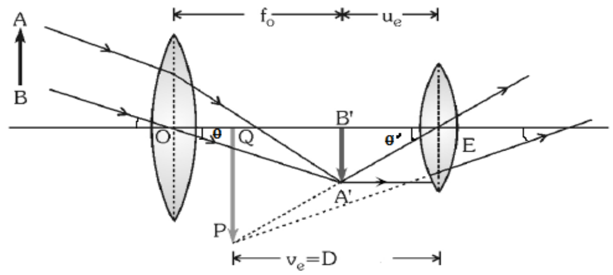
$= 100\left[\frac{1}{25} + \frac{1}{5}\right]$

$= 100 \times \frac{6}{25}$

$= 24$

Magnifying power of refracting telescope when final image is at near point

The arrangement is shown in figure below. The separation of the lenses is less than when the final image is formed at infinity. The intermediate image, though still in the focal plane of the objective, is now inside the focal point (Fe) of the eyepiece lens and in such a position that the final image is at the near point.



The angular magnification is defined as the ratio of the subtended at eye by the final image to the angle subtended at unaided eye by the object.

$$M = \frac{\beta}{\alpha}$$

Where

$\alpha = \frac{h}{f_o}$, and $\beta = \frac{h}{u_e}$

$\tan\beta \approx \beta$, and $\tan\alpha \approx \alpha$

Therefore, since $M = \frac{\beta}{\alpha}$

$$= \frac{h \div u_e}{h \div f_o}$$

$$M = \frac{f_o}{u_e}$$

From the lens formula, $\frac{1}{f_e} = \frac{1}{u_e} - \frac{1}{v_e}$, (for the virtual image)

And also $v_e = D$, then

$$\frac{1}{f_e} = \frac{1}{u_e} - \frac{1}{D} \rightarrow \frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{D}$$

$u_e = \frac{f_e \times D}{f_e + D}$, $M = \dfrac{f_0}{\frac{f_e \times D}{f_e + D}} = \dfrac{f_0(f_e + D)}{f_e \times D}$

$M = f_o\left(\frac{1}{D} + \frac{1}{f_e}\right)$, multiply and divide by f_e we get $M = \frac{f_o}{f_e}\left(\frac{f_e}{D} + \frac{f_e}{f_e}\right)$

Therefore $M = \frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$

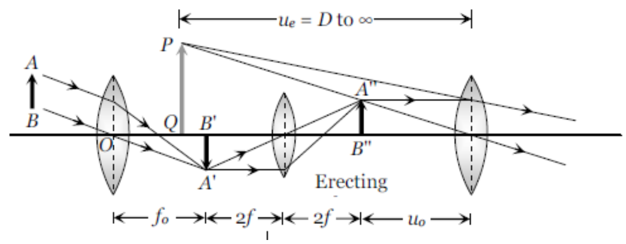
Length of the tube $L_D = f_o + u_e = f_o + \frac{f_e \times D}{f_e + D} (2)$

2. Other refracting telescopes

(a)Terrestrial telescope

- Used to see far off object on the earth.
- It consists of three converging lens: objective, eye lens and erecting lens.
- It's final image is virtual erect and magnified.
- Angular magnification $M = \frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$ at near point
- Angular magnification of image at infinity

$$\boldsymbol{M = \frac{f_o}{f_e}}$$



- Length of the tube when the image is at near point:

$$L_D = f_o + 4f + u_e$$

$$= f_o + 4f + \frac{f_e \times D}{f_e + D}$$

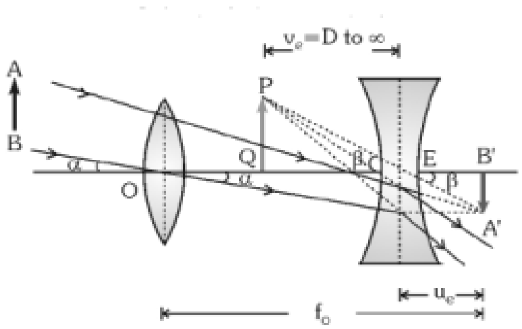
And at infinity $L_\infty = f_o + 4f + f_e$

(b) Galilean telescope

- It is also a terrestrial telescope but of much smaller field of view.
- Objective is a converging lens while eyepiece lens is diverging lens.
- Magnification of image at near point:

$M_D = \frac{f_o}{f_e}\left(1 - \frac{f_e}{D}\right)$, and image at infinity: $M_\infty = \frac{f_o}{f_e}$

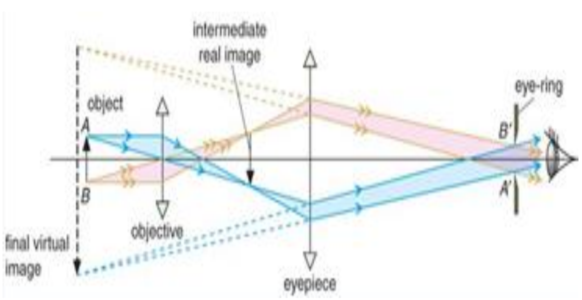
- Length of the tube when image formed at near point: $L_D = f_o - u_e$, when image is at infinity the length of the tube is: $L_\infty = f_o - f_e$



3. Eye- ring

The eye ring is the best position to place the eye in order to be able to view as much of the final as possible.

In case of telescope, all the light from a distant must pass though the eye ring after leaving the telescope. So by placing the eye at the eye ring, the viewer is able to see the final image as much possible.



A&B is a small region through which all the light passes objective lens and the eye piece lens are made to pass. Hence observed by the eye pupil at A , B and the pupil must have diameter equal to AB.

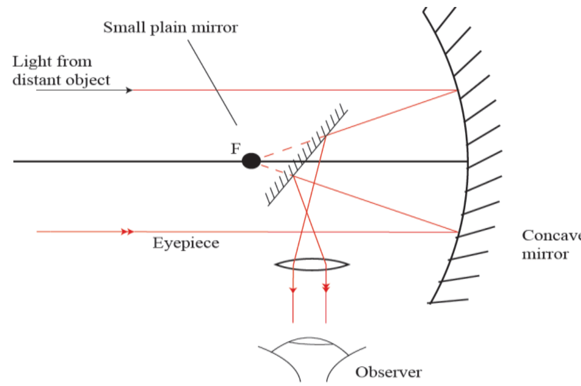
Reflecting telescopes

Reflecting telescopes consist of a large concave mirror of long focal length as their objective.

Note: The reflecting telescopes are free from chromatic aberration since no refraction occurs.

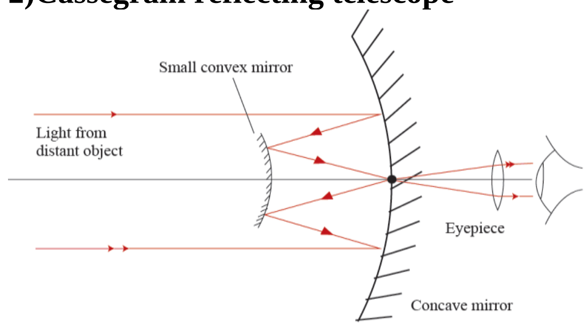
There are three kinds of reflector telescopes, all named after their inventors.

1).*The Newtonian reflecting telescope*



The Newtonian telescope is commonly used by amateur astronomers. A small plane mirror is used to direct the light from the concave mirror, which acts as an objective into an eye piece.

2)Cassegrain reflecting telescope



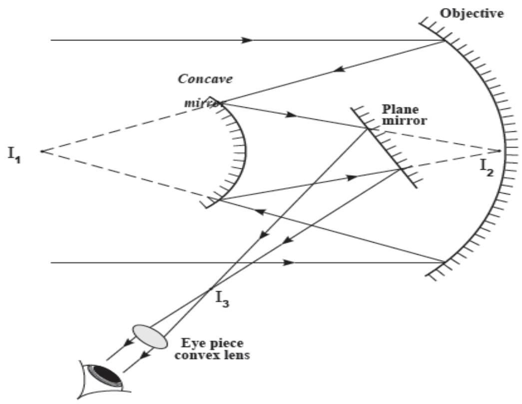
This is the type used in most observatories

It consists of a concave mirror which acts as an objective, a small convex mirror and the eye piece lens. Light from a distant object is reflected by the concave mirror to the convex mirror which reflects it back to the centre of the concave mirror where there is
a small hole to allow the light through. So the convex mirror forms the final image (real) at the pole of the objective.

3) Coude Reflector Telescope

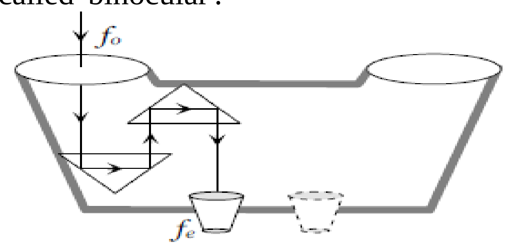
This is a combination of Newtonian and cassegrain reflector telescopes.

It is consist of a plane and convex mirrors used in reflecting telescopes are used to bring the light to a more convenient focus where the image can be photographed and magnified several times by the eye piece for observation.



Prism Binocular

If two telescopes are mounted parallel to each other so that an object can be seen by both the eyes simultaneously, the arrangement is called 'binocular'.



In a binocular, the length of each tube is reduced by using a set of totally reflecting prisms which provided intense, erect image free from lateral inversion. Through a binocular we get two images of the same object from different angles at same time.

Difference between compound microscope and telescope

Compound microscope	**Astronomical telescope**
Objective lens has smaller focal length, than the eyepiece	Objective lens has larger focal length than the eyepiece
Distance between the objective lens and the eyepiece is greater than f_0+f_e	Distance between the objective lens and the eyepiece is equal to f_0+f_e
It is used to see very small objects	It is used to see distant astronomical objects

Light gathering: the ability of telescope to collect a lot more light than the human eye, its light gathering power is probably its most important feature.

Resolving power: object that are so close together in the sky that they blur together into a single blob are easily seen as separate object with a good telescope. Or Resolving power is the ablity of optical instruments to form a clear image.

Exercises

1.A certain eye can focus only on objects closer than 50.0 cm. What word characterizes this type of vision problem? What sort of contact lens (described both in focal length and power) will correct this problem?

2.A patient's eye can focus only on objects beyond 100 cm. What word characterizes this type of vision problem? What are the focal length and power of the contact lens needed to correct this problem?

3.A distant object is viewed with a relaxed eye with the help of a small Galilean telescope having an objective of focal length 15 cm and an eye piece of focal length 3 cm

(A) The distance between the objective and the eyepiece lens is 12 cm.

(B) The angular magnification of object is 5

(C) Image of the object is erect

(D) The distance between objective and eye piece lens is 18 cm

5.A microscope consists of an objective with a focal length 2 mm and an eye piece with a focal length 40 mm. The distance between the foci (which are between the lenses) of objective and eyepiece is 18 cm. The total magnification of the microscope is(Consider normal adjustment and take D = 25 cm)

(A) 562.5 (B) 625 (C) 265 (D) 62.5

6.An astronomical telescope has its two lenses spaced 75.2 cm apart. If the objective lens has a focal length of 74.5 cm, what is the magnification of this telescope? Assume a relaxed eye

7.An astronomical telescope has an angular magnification of magnitude 5 for distant objects. The separation between the objective and the eyepiece is 36 cm. The Final image is formed at infinity. The focal length fo of the objective and fe of the eyepiece are
(A) 45 cm and – 9 cm respectively (B) 50 cm and 10 cm respectively
(C) 7.2 cm and 5 cm respectively (D) 30 cm and 6 cm respectively

8.A single converging lens used as a simple microscope. In the position of maximum angular magnification,
(A) the object is placed at the focus of the lens
(B) the object is placed between the lens and its focus
(C) the image is formed at infinity
(D) the object and the image subtend the same angle at the eye.

9.In a reflecting astronomical telescope, if the objective (a spherical mirror) is replaced by a parabolic mirror of the same focal length and aperture, then
(A) The final image will be erect
(B) The larger image will be obtained
(C) The telescope will gather more light
(D) Spherical aberration will be absent

10.The focal lengths of the objective and the eyepiece of a compound microscope are 2.0 cm and 3.0 cm respectively. The distance between the objective and the eyepiece is 15.0 cm. The final image formed by the eyepiece is at infinity. Find the distance of object and image produced by the objective, from the objective lens.

11.An 8cm focal-length converging lens is used as a "jeweler's loupe", which is a magnifying glass. Estimate (a) the magnification when the eye is relaxed, and (b) the magnification if the eye is focused at its near point N=25cm.

12.An astronomical telescope has an objective of focal length 200 cm and an eyepiece of focal length 4.0 cm, the telescope is focused to see an object 10 km from the objective. The final image is formed at infinity. Find the length of the tube and the angular magnification produced by the telescope.

13.A microscope has a magnifying power of 600, and an eyepiece of angular magnification of 15. The objective lens is 22 cm from the eyepiece. Without making any approximations, calculate (a) the focal length of the eyepiece, (b) the location of the object such that it is in focus for a normal relaxed eye, and (c) the focal length of the objective lens.

TOPIC AREA: MECHANICS

Unit 3. Moments and equilibrium

3.1. Scalar and vector quantities

When defining the parameters used in describing motion, it was noted that some of them are vector quantities and others are scalar quantities.

1. **A scalar quantity.**

A **scalar quantity** is a quantity that can be completely specified by its magnitude (i.e size) together with the appropriate units.

e.g: The temperature of boiling water is completely specified by giving its **magnitude** and **units**; 100^0c or 373 K.

Here are some examples of scalar quantities: temperature, density, energy, length, area, volume time, angle, speed, electric charge power, energy, resistance, pressure, potential difference, frequency and wavelength.

2. **Vector quantity**

A vector quantity is a quantity that is completely specified by its **magnitude** and **direction** together with the appropriate units.

As example, when an object moves, we are interested to know how fast and where it is moving; we want to know the **speed** and **direction** of the motion. The quantity that tells us this is the **velocity** and it is the vector.

Some common important examples of vector quantities we deal with are: **displacement, velocity, acceleration, force, torque, linear momentum, and angular momentum.**

The magnitude of **displacement vector** is the **distance** and for the **velocity** is the **speed.** A vector quantity is usually represented using an arrow above its head.

Example: -Force, $(\vec{F})$; -momentum, $\vec{p}$; velocity, $\vec{v}$ Etc.

3. 2. Force as vector

Force is a pull or push on an object. It can affect the object by **changing** its **shapes** and its **motion.**

Among the types of force, there are **Friction**, **weight**, **contact**, **magnetic, electrostatic** and **elastic force**.

Force is a **vector** quantity. This means it has both magnitude (size) and direction.

Its SI unit is **Newton**.

Test your self

1. .... is an example of a scalar quantity
   a) Velocity.
   b) Force.
   c) Volume.
   d) Acceleration.
2. .... is an example of a vector quantity
   a) Mass.
   b) Force.
   c) Volume.
   d) Density.
3. A scalar quantity:
   a) Always has mass.
   b) is a quantity that is completely specified by its magnitude.
   c) Shows direction.
   d) Does not have units.
4. A vector quantity.
   a) Can be a dimensionless quantity.
   b) Specifies only magnitude.
   c) Specifies only direction.
   d) Specifies both a magnitude and a direction.
5. A boy pushes against the wall with 50 kilogrammes of force. The wall does not move. The resultant force is:
   a) -50 kilogrammes.
   b) 100 kilogrammes.
   c) 0 kilogrammes.
   d) -75 kilogrammes.

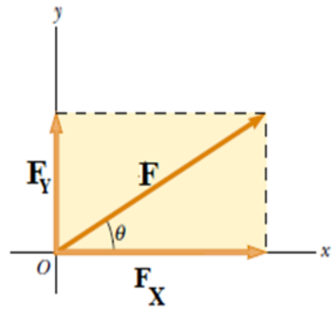
6. A man walks 3 miles north then turns right and walks 4 miles east. The resultant displacement is:
    a) 1 kilometer SW
    b) 7 kilometers NE
    c) 5 kilometers NE
    d) 5 kilometers E

7. The difference between speed and velocity is:
    a) Speed has no units.
    b) Speed shows only magnitude, while velocity represents both
    c) magnitude (strength) and direction.
    d) c) They use different units to represent their magnitude.
    e) d) Velocity has a higher magnitude.
8. The resultant magnitude of two vectors
    a) Is always positive.
    b) Can never be zero.
    c) Can never be negative.
    d) Is usually zero.
9. Which of the following is not true
    a) Velocity can be negative.
    b) Velocity is a vector.
    c) Speed is a scalar.
    d) Speed can be negative.

3.3. Components of a vector force and unit vectors

1. Components of a vector force

Consider a vector F lying in the *xy* plane and making an arbitrary angle θ with the positive *x* axis, as shown in Figure below



The *x* component $\boldsymbol{F_x}$ of the vector **F** is equal to the projection of **F** along the *x* axis of a coordinate system, as shown in Figure below, where $F_x = F\cos\theta$.

The *y* component $\boldsymbol{F_y}$ of **F** is the projection of **F** along the *y* axis, where $F_y = F\sin\theta$

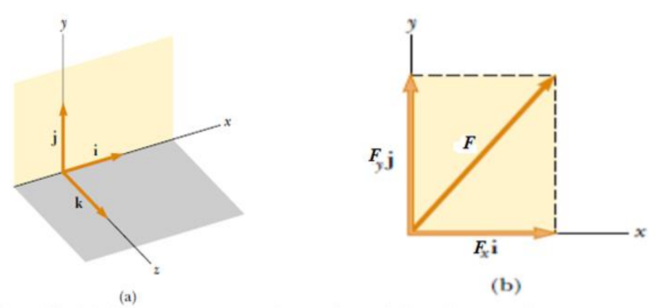
2. Unit vectors

A unit vector is a dimensionless vector having a magnitude of exactly **1**.

Vector quantities often are expressed in terms of unit vectors.

We shall use the symbols**, i, j,** and **k** to represent unit vectors pointing in the positive ***x*, *y*,** and ***z*** directions, respectively.

The unit vectors are mutually perpendicular vectors in a right-handed coordinate system, as shown in Figure below. The magnitude of each unit vector equals **1;** that is, $|i| = |j| = |k| = 1$



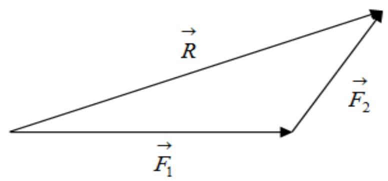
3.4. Addition of forces 'resultant force'

A resultant force is a single force which can replace two or more forces and produce the same effect on the body as the forces.

We can add two vector forces graphically, using either the **triangle** method or the **parallelogram** rule

1. Triangle addition law of forces

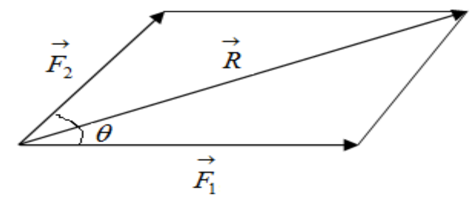
It states that if two forces acting simultaneously on a body are represented in magnitude and the direction by the two sides of triangle taken in order then their resultant may be represented in magnitude and the direction by the third side taken in opposite order.



The resultant force $\vec{R} = \vec{F_1} + \vec{F_2}$ When vectors and are represented as above so that the tail of vector F_2 is connected to the head of vector F_1, the magnitude of the resultant is given by $R = \sqrt{F_1^2 + F_2^2 - 2F_1F_2 \cos\theta}$ **. If** the **tai**l of the second force is at the **head** of the first force

2. Parallelogram law of forces

It states that if two forces, acting simultaneously on a particle, be represented in magnitude parallelogram then their resultant may be represented in magnitude and the direction by the diagonal of the parallelogram which passes through their point of intersection

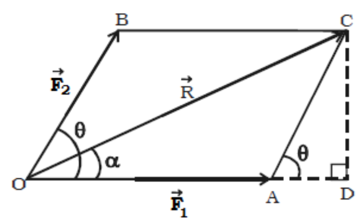


$\vec{R} = \vec{F_1} + \vec{F_2}$

If θ is an angle between $\vec{F_1}$ and $\vec{F_2}$

Then the magnitude of $\vec{R}$ is $R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos\theta}$.

Proof



From right angled triangle *OCD*,
$OC^2 = OD^2 + CD^2$
$= (OA + AD)^2 + CD2$
$= OA^2 + AD^2 + 2.OA.AD + CD^2$ *(1)*

From right angled Δ CAD,
$AC^2 = AD^2 + CD^2$ (2)

Substituting (2) in (1)
$OC^2 = OA^2 + AC^2 + 2OA.AD$ *(3)*

From right angled Δ CAD,
$CD = AC \sin\theta$ (4)
$AD = AC \cos\theta$ (5)

Substituting (5) in (3) $OC^2 = OA^2 + AC^2 + 2\ OA.AC \cos\theta$
Substituting $OC = R$, $OA = F_1$, $OB = AC = F_2$ in equation (3)

$$R^2 = F_1^2 + F_2^2 + 2F_1F_2 \cos\theta$$

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos\theta}$$

1. Algebraic method

It is easier to find the resultant of a set of forces if the forces are expressed in terms of Cartesian components or resolutes. Forces are manipulated as vectors.

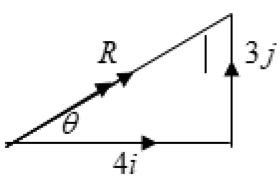
Examples: Three forces $(2i - j)N$, $3iN$ and $(-i + 4j)N$ where i and j are unit vectors due East and due North respectively, act on a particle. Find the magnitude and direction of the resultant force.

Solution: $\vec{R} = (2i - j) + 3i + (-i + 4j)$

$$\vec{R} = 2i + 3i - i + (-j) + 4j$$

$$\vec{R} = 4i + 3j$$

The magnitude of the resultant is $\|\vec{R}\| = \sqrt{4^2 + 3^2} = 5$



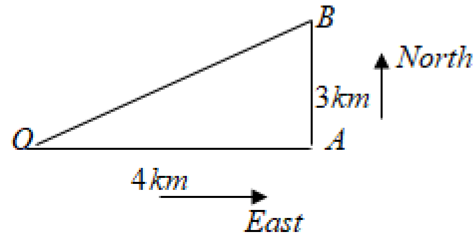
$\tan\theta = \frac{3}{4}$

Hence $\theta = 36.869^0$. The resultant of these forces is therefore of magnitude $5N$ and acts at an angle of 36.869^0 to unit vector i .

Examples

1. A man starts at point O and walks $4km$ East to the point A and then $3km$ North to the point B. Find the distance that he walked, the final displacement from O and describe the direction of this displacement.

Solution: let draw a diagram representing his journey



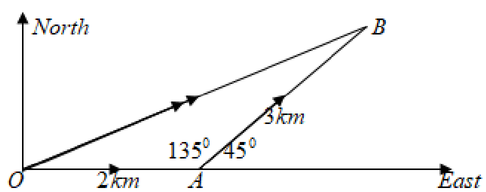
The displacement is given by Pythagoras's theorem $R = 4^2 + 3^2 = \sqrt{25} = 5km$

***Direction is** given by the angle between the horizontal axis and the displacement*

$\tan\theta = \frac{\overline{AB}}{\overline{OA}} = \frac{3}{4}$ The direction $\theta = \tan^{-1}\frac{3}{4} = 36.86^0$ in the *North East* .

2. A man walks $2km$ due East from O to A and then $3km$ in a North-East direction from A to B. Find the distance of B from O, and describe the displacement $\overrightarrow{OB}$.

Solution: diagram of the journey



Using cosine rule for a triangle

3.5. Moment of a force (0r torque, τ) about a point $c = \sqrt{a^2 + b^2 - 2ab\cos\hat{C}}$

$R = \sqrt{2^{2+}3^2 - 2x2x3\cos 135} = \sqrt{21.48} = 4.64\,km$

The direction of this displacement is given by sine rule

General, as $\frac{a}{\sin\hat{A}} = \frac{b}{\sin\hat{B}} = \frac{c}{\sin\hat{C}}$

Let take $\frac{OB}{\sin 135^0} = \frac{AB}{\sin\theta}$ i.e.

$$\sin\theta = \frac{AB\sin 135^0}{OB}$$

$$\sin\theta = \frac{3\sin 135^0}{4.64}$$

$\theta \approx 27.2^0$ to the East

3.5. Moment of a force (0r torque, τ) about a point

3.5.0. Introduction

Why are a door's doorknob (door handle) and hinges (joint that holds two parts together so that one can swing relative to another part) placed near opposite edges of the door?

Answer:

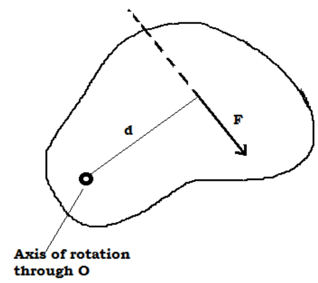
If you want to open a heavy door, you must certainly apply a force.

The farther we are from the hinges, the easier we close or open the door.

3.5.1. Definition of torque and its mathematical calculation

The tendency of a force to rotate an object about some axis is measured by a vector quantity **called moment of force or torque** τ .

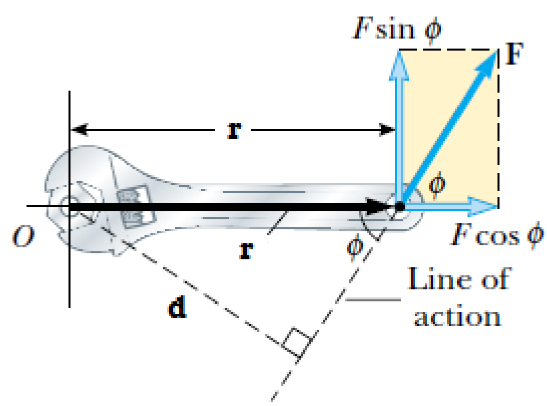
Torque or turning moment of force is the product of a force and the lever arm, where the lever arm is the perpendicular distance from the axis to the line of application of the force.



$\tau = Fd$

Torque has unit of Newton-meter **[Nm].**

The figure below shows that the force F has a greater rotating tendency about O, as F increases and as the moment arm d increases. The component $F\sin\phi$ tends to rotate the wrench about O.

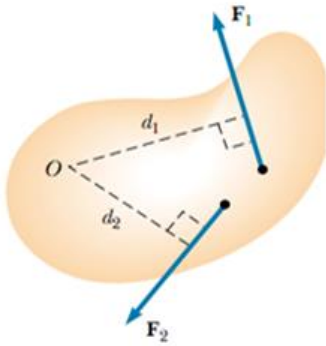


Here we define torque as $\tau = rF\sin\phi = Fd$

r is the distance between the pivot point and the point of application of F and d is the perpendicular distance from the pivot point to the line of action of F. (The *line of action* of a force is an imaginary line extending out both ends of the vector representing the force).

3.5.2. Torque made by two or many forces

In this example below, F_2 tends to rotate the object clockwise, and F_1 tends to rotate it counterclockwise.

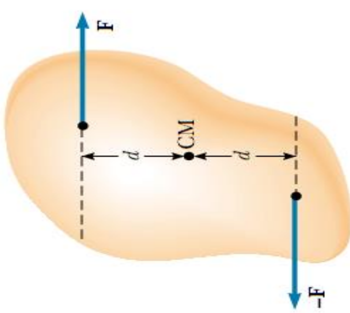


Hence, the net torque about O is $\sum \tau = \tau_1 + \tau_2 = F_1 d_1 - F_2 d_2$

- Do not confuse torque and work, which have the same units but are very different concepts.

3.5.3. Couples and torques

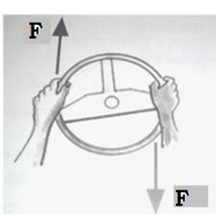
A **couple** consists of two forces of equal magnitude acting in opposite directions along parallel lines of action. Their lines of action do not coincide.



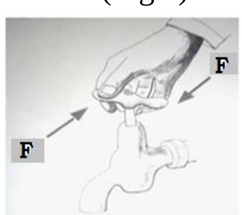
Other examples

-Couple of force on steering wheel fig.(a).

-Turning a water tap, one of your fingers pushes on one end and another end (Fig.b)



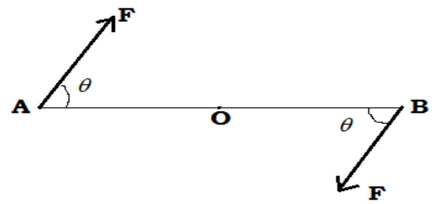
(a)Couple of force in steering wheel



(b)Couple of force on tap water

3.5.4. Moment of a couple

The torque or moment of a couple is a measure of its ability to rotate a body on which it is acting.



$$C = \tau_1 + \tau_2 = F \times \vec{AO} \sin\theta + F \times \vec{OB} \sin\theta$$

$$C = \tau_1 + \tau_2 = (\vec{AO} + \vec{OB}) F \sin\theta = F\ \vec{AB} \sin\theta$$

Thus the torque about O does not depend on the position of O .

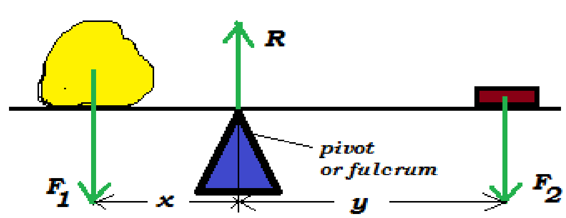
3.6. Force and levers, Law of the lever

A lever is a beam connected to ground by a hinge or pivot called a fulcrum.

It is a movable bar that pivots on a fulcrum attached to a fixed point.

Law of the lever

The ideal lever does not dissipate or store energy, which means there is no friction in the hinge or bending in the beam. In this case, the power into the lever equals the power out, and the ratio of output to input force is given by the ratio of the distances from the fulcrum to the points of application of these forces. This is known as the ***law of the lever****.*



From the principle of levers

$F_1.x = F_2.y$

where F_1 is the input force to the lever and F_2 is the output force. The distances x and y are the perpendicular distances between the forces and the fulcrum.

The ratio of the distances $\frac{x}{y}$ also called velocity ratio of two masses is equal to the ratio of the output force (F_2) to the input force (F_1), or mechanical advantage MA, then

$$MA = \frac{F_2}{F_1} = \frac{x}{y}$$

This is the *law of the lever*, which was proven by Archimedes **.**

The upward reaction force on the pivot $R = F_1 + F_2$

Principle of moment

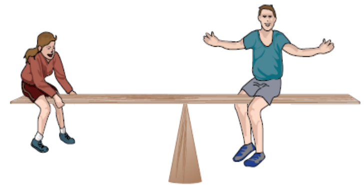
Principle of moment state that "the ***sum of the clockwise moments about any point equals the sum of the anticlockwise moments about the same point, for a body under equilibrium condition".***

Example1

A woman of mass $m = 55kg$ sits on the left end of a seesaw a plank of length L =4.00 m, pivoted in the middle as in Figure below:

(a) Where should a man of mass M =75.0 kg sit if the system (see-saw plus man and woman) is to be balanced?

(b) Find the normal force exerted by the pivot if the plank has a mass of $m_{pl} = 120.0kg$.



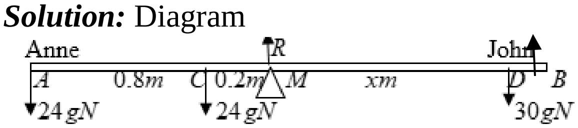
Equilibrium of a Non-uniform rod, Reaction at the pivot and moments

The centre of mass of a non-uniform rod is located at some point other than the mid-point of the rod.

Example 1:

Two children are sitting on a non-uniform plank AB of mass $24kg$ and length $2m$. This plank is pivoted at M the mid-point of AB. Anne has mass $24kg$ and sits at A and John has mass $30kg$. Find where John must sit for the plank to be horizontal if the centre of mass is at $0.8m$ from A.

Solution: Diagram



If you take the moments about M, the unknown force at the pivot is not required.

Momentsabout M

$$24g \times 1 + 24g \times 0.2 = 30gx$$

$$24g + 4.8g = 30gx$$

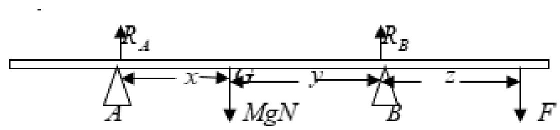
$$28.8g = 30gx$$

$$x = \frac{28.8g}{30g} \quad x = 0.96m$$

John should sit at $0.96m$ from M, or at $1m - 0.96m = 0.04m$ from B.

Tilting

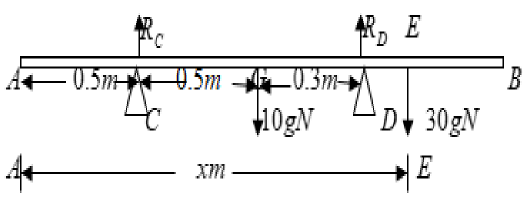
A heavy rod of mass M placed across two supports at A and B which are at the same horizontal level will be in equilibrium provided its centre of mass G is between A and B as shown below



If the magnitude of the extra-force is such that the rod is still in equilibrium but the reaction at A ***has decreased to zero, the rod is said to be on the point of tilting (or turning) about*** B

Example:1 A uniform plank AB of mass $10kg$ and length $2m$ is in equilibrium in a horizontal position resting on two supports at C and D where $AC = 0.5m$ and $AD = 1.3m$. A boy of mass $30kg$ stands on the plank at point E and the plank is on the point of tilting about D. By modeling the plank as a uniform rod and the boy as a particle, calculate the distance AE.

Solution: Diagram:



Let the distance AE be xm

As the rod is on the point of tilting about D, the reaction at C is zero i.e. $R_C = 0$

The distance $DE = x - 1.3$. Lets take moment at D

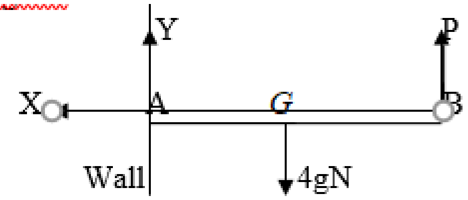
$30g(x-1.3) = 10g \times 0.3 \Rightarrow x = 0.1 + 1.3 = 1.4m$

$x = 1.4$. The distance AE is $1.4m$.

FREE HINGED UNIFORM ROD

Example1

A uniform rod AB of mass 4kg and length 80cm is freely hinged to a vertical wall. A force P, as shown in the diagram, is applied at the point B and keeps the rod horizontal and in equilibrium. The forces X and Y are the horizontal and vertical components of the reaction at the hinge. Find the magnitudes of the forces X, Y and P



Solution.

Take moments about A:

M (A): $P \times 80 = 4g \times 40$

$P = 2g$ or $P = 19.6N$

Resolving vertically gives:

$P\cos 0^0 + Y\cos 0^0 = 4g$

$Y = 4g - p = 4g - 2g \Rightarrow Y = 2g = 19.6N$

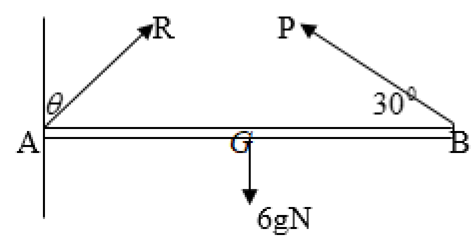
Resolving horizontally gives: $X = 0$

The force $X = 0$; $Y = 19.6N$ and $P = 19.6N$

<u>Reaction at the hinge</u> (making an angle with the vertical).

Example 2

A uniform rod AB of mass 6kg and length 4m is freely hinged at A to a vertical wall. The force P applied at B as shown in the diagram, keeps the rod horizontal and in equilibrium; R is the force of reaction at the hinge and θ is the angle that the line of action of this force makes with the vertical. Find the magnitude of the forces P and R and the angle θ.



Solution: Take the moments about A:

M (A): $6g \times 2 = P \times 4\sin 30^0$

$12g = 2p$ *hence* $p = 6g = 58.8N$

Resolving vertically:

$R\cos\theta + P\cos 60^0 = 6g$

Substituting for P gives:

$R\cos\theta + 6g \times 0.5 = 6g$

$R\cos\theta = 3g$ (1)

Resolving horizontally gives:

$R\sin\theta = P\cos 30^0$ but P=6g

$R\sin\theta = 6g\cos 30^0$ (2)

Dividing eqn (2) by eqn (1) gives:

$$\frac{R\sin\theta}{R\cos\theta} = \frac{6g\cos 30^0}{3g}$$

$\tan\theta = \sqrt{3}$ or $\theta = 60^0$ From $R\cos 60^0 = 3g$

$$R = \frac{3g}{\cos 60^0} = 3g \times 2 \text{ hence } R = 6g$$

The forces $P = 6gN$, $R = 6gN$ and the angle $\theta = 60^0$.

3.7. Equilibrium

3.7.0. Definition

When all the forces that act upon an object are balanced, then the object is said to be in a state of equilibrium. The term **equilibrium** implies either that the object is at rest or that its center of mass moves with constant velocity. For that reason there are 2 main categories of equilibrium **static equilibrium and dynamic equilibrium**. An object in equilibrium is either:

1. at rest and staying at rest, or
2. In motion and continuing in motion with the same speed and direction.

Static equilibrium: This is when the combined effect of all the forces acting on a body is zero and the body is in the state of rest. Example: a block lying on a table.

Dynamic equilibrium: This is when a body is in state of uniform motion and the resultant of all forces acting upon it is zero. Example: jump by using parachute.

Static equilibrium

Static means stationary or at rest. Statics is the study of a particle at rest i.e. the particle which is not in motion.

If all the forces acting on a body meet at a point, they are called **concurrent forces**; if they do not meet they are called **non concurrent forces**. Concurrent forces are **non parallel forces**.

3.7.1. Types of equilibrium: (Stable, unstable and neutral)

1. **Stable equilibrium**

The body is **in stable equilibrium** when it returns to its original position after being slightly disturbed. The body has the board base hence a lower centre of gravity. The body does not topple.

A cone balancing on its base, on a horizontal surface

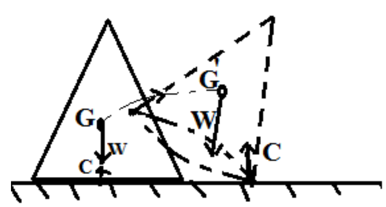


Fig A : Stable equilibrium

2. **Unstable equilibrium**

An object is in **unstable equilibrium** when does not return to its original position after being slightly disturbed. The body has a **narrow base** and high centre of gravity. thus causing it to topple

A cone balancing on its point, or apex, on a horizontal surface

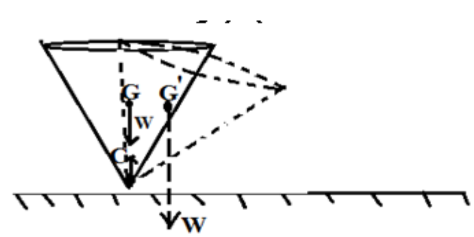
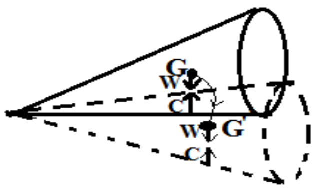


Fig B: Unstable equilibrium

3. **Neutral equilibrium**

An object is in **neutral equilibrium** when it moves to a new position when it is disturbed. It does neither move back, to neither its original position nor any further. This is a typical equilibrium of spherical bodies. The centre of gravity is exactly at the centre and does not change its position.



(C) Neutral equilibrium

A cone that can role on its edge.

3.7.2. Types of forces involved in equilibrium

In general, forces are classified into two main categories: **contact forces** and **non contact forces**. As the equilibrium results in forces acting on a particle, we will consider **contact forces**.

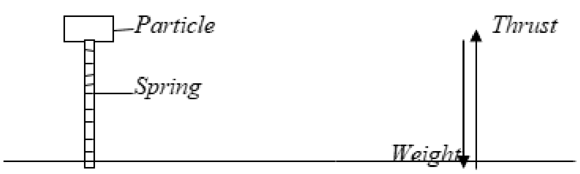
a) ***Weight*** W

.

b) ***Tension*** T in a string for a hanged body $(W = mg) = T$

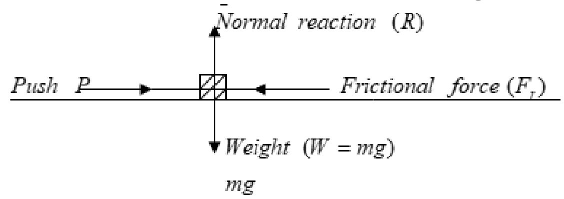
c) ***Thrust***

Now, imagine a similar situation where a particle is supported by a vertical spring from below.. This upwards force is called the thrust.



d. ***Friction and coefficient of static friction***

Let illustrate the situation on the diagram.



If a horizontal force P is applied to the particle, it does not necessary move because there is a force opposing the force P; this opposing force is called ***frictional force***, and it is denoted by F_r. If P and F_r are the only horizontal forces acting on the particle, then so longer as the particle is stationary $P = F_r$. The magnitude of the frictional force is just sufficient to prevent the relative motion. The frictional force for a particular surface is not constant; it increases as the applied force increases until the force F_r reaches a value $F_{r\max}$ beyond which it cannot increase. The particle is then just about to move, and it is said to be in a state of ***limiting equilibrium***. At this point, friction is said to be limiting. But $F_{r\max} = \mu R$ where μ is the constant of proportionality called the ***coefficient of friction*** (μ is read mu). The range of the frictional force is given by $0 \prec F_r \leq \mu R$. When the particle is still at rest and in contact with the surface, the coefficient of friction is called the ***coefficient of static friction.***

In summary,

1.Frictional force acts to oppose the relative motion

2.Until it reaches its limiting value, the magnitude of the frictional force is just sufficient to prevent the relative motion

3.When the limiting value is reached, $F_r = \mu R$, where R is the normal contact force and μ is the coefficient of static friction

4. For all rough surfaces, $0 \prec F_r \leq \mu R$

5. For all smooth surfaces, $F_r = 0$

6.When the particle begins to slide, the frictional force takes its limiting value μR and acts in the direction opposite to the direction of relative motion.

3 .7. 2. Condition for equilibrium of a body about an axis

We now have two necessary conditions for equilibrium of an object:

-The resultant external force must be equal zero (The components of forces in

1.The sum of all forces in the x-direction or horizontal is zero

$$\Sigma F_x = 0$$

2. The sum of all forces in the y-direction or vertical is zero

$$\Sigma F_y = o$$

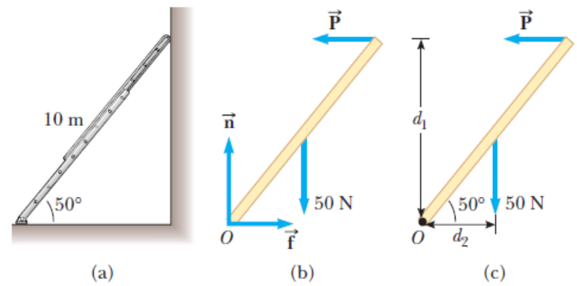
3.The resultant external torque about any axis must be zero (The sum of clockwise moments about any point equals the sum of the anticlockwise moments about the same point) $\sum \tau = 0$.

1. Ladder problems

The situation of a ladder resting against a wall, gives rise to a variety of problems. The wall may be rough or smooth, as also may the ground. The ground may, or may not, be horizontal. It should be remembered that where the ladder rests against a smooth surface, there will only be a normal reaction R at that point. When the surfaces in contact are rough, there is also a frictional force F_r which acts parallel to the surfaces in contact, and in a direction opposite to that in which the ladder would move

Example

A uniform ladder 10.0 m long and weighing 50.0 N rests against a smooth vertical wall as in Figure below. If the ladder is just on the verge of slipping when it makes a 50^0 angle with the ground, find the coefficient of static friction between the ladder and ground.



Solution

Apply the first condition of equilibrium to the ladder:

$\Sigma F_x = f - P = 0 \rightarrow f = P$

$\Sigma F_y = n - mg = 0$

$n - 50.0 = 0 \rightarrow n = 50.0N$

Taking the moments of force about O

$\Sigma \tau_i = \tau_f + \tau_n + \tau_{grav} + \tau_p = 0$

Apply the second condition of equilibrium, computing torques around the base of the ladder, with τ_{grav} standing for the torque due to the ladder's 50.0N weight:

$\tau_f + \tau_n + (mg)(\frac{L}{2}) sin40^0 + PLsin50^0 = 0$

$0 + 0 - (50.0N)(5.00m) sin40^0 + P(10.0m) sin50.0^0 = 0$

$P = 21.0N$

we now have $f = \mu_s n: = P = 21.0N$.

$21.0N = \mu_s (50.0N)$

$$\mu_s = \frac{21.0N}{50.0N} = 0.420$$

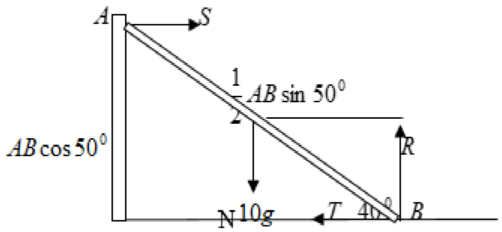
Example 2

A uniform ladder AB, of mass 10kg and length 4m, rests with its upper end A against a smooth vertical wall and end B on smooth horizontal ground. A light horizontal string, which has one end attached to B and the other end attached to the wall, keeps the ladder in equilibrium inclined at 40^0 to the horizontal.

The vertical plane containing the ladder and the string is at right angles to the wall. Find the tension T in the string and the normal reactions at the points A and B.

Solution:

The diagram shows the forces acting on the ladder



Resolving vertically: $R = 10g$ then $R = 98N$

Resolving horizontally: $T = S$ Take the moments about B:

M (B): $S \times 4\sin 40^0 = 10g \times 2\cos 40^0$

$$S = \frac{5g\cos 40^0}{\sin 40^0} \quad S = 58.4N$$

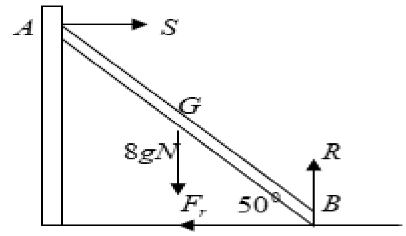
This follows that $T = 58.4N$

Rough contact at foot of ladder

If the ladder rests on ground which is rough, then there will be a frictional force F_r acting on the ladder at this point. The maximum value of this frictional force depends upon the roughness of the contact between the ladder and the ground.

Example 3

The diagram shows a ladder AB of mass 8kg and length 6m resting in equilibrium at an angle of 50^0 to the horizontal with its upper end A against a smooth vertical wall and its lower end B on a rough horizontal ground, coefficient of friction μ. Find the forces S, F_r and R and the least possible value of μ if the centre of gravity G of the ladder is 2m from B.



Solution:

Take moments about B

$S \times 6\sin 50^0 = 8g \times 2\cos 50^0$

$S = \frac{16g\cos 50^0}{6\sin 50^0}$ i.e $S = 21.9N$

Horizontally: $F_r = S$ Hence $F_r = 21.9N$

Vertically: $R = 8g$ then $R = 78.4N$

Since $\mu = \frac{F_{r\max}}{R}$, μ must be at least $\frac{21.9}{78.4} = 0.28$ .Theforce $S = 21.9N$, $F_r = 21.9N$, $R = 78.4N$ and μ must be at least 0.28

Climbing a ladder

Whether or not it is safe to ascend to the top of a ladder will depend upon the magnitude of the frictional force which acts on the foot of the ladder. This will depend upon the roughness of the ground on which the ladder rests.

If the ladder is found to be in limiting equilibrium when a person is part way up a ladder, then any further ascent will cause the ladder to slip. To determine how far a ladder may be ascended, consider the situation when the climber is at a distance s up the ladder and the ladder is in limiting equilibrium. The following example illustrates the method.

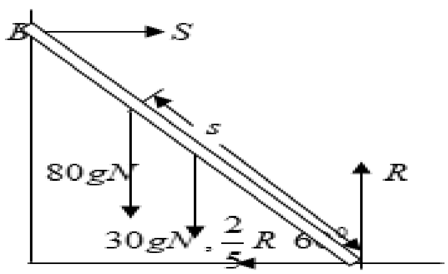
Example 4

A uniform ladder of mass 30kg and length 5m rests against a smooth vertical wall with its lower end on rough ground, coefficient of friction $\frac{2}{5}$. The ladder is inclined at 60^0 to the horizontal. Find how far a man of mass 80kg can ascend the ladder without it slipping.

Solution:

Assume the man can ascend a distance s from the foot of the ladder, which is then in limiting equilibrium. The maximum frictional force μR will then act at the foot of the ladder.

The forces acting on the ladder are then as shown on the diagram:



Vertically: $R = 30g + 80g = 110g$ (1)

Horizontally: $S = \frac{2}{5}R$ (2)

Take moments about A:

$$30g \; x \frac{5}{2}\cos 60 + 80g \; x \; s\cos 60^0 = S \; x \; 5\sin 60^0$$

(3)

From equations (1) and (2):

$$S = \frac{2}{5} \times 110g = 44g$$

Substituting for S in equation (3) gives:

$$\frac{75g}{2} + 40gs = 44g \times 5\sin 60^0$$

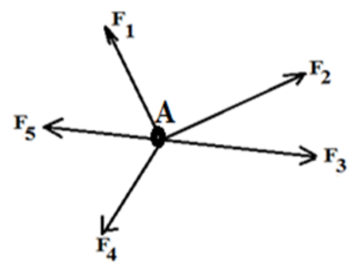
That is $s = 3.83m$

The man can climb $3.83m$ up the ladder, at which point the ladder will be on the point of slipping.

Other examples of equilibrium of bodies

1. Coplanar concurrent forces on a body in equilibrium

Coplanar concurrent forces are forces whose lines of action pass through a common point and the forces have different directions in the same plane.



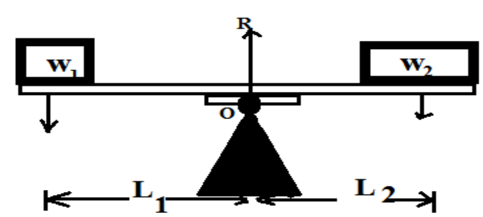
When several forces act on a particle, the particle is said to be in equilibrium if there is no unbalanced forces acting on it.

$$\sum \vec{F_y} = 0$$

$$\sum \vec{F_x} = 0$$

2. Seesaw

Consider a seesaw with three parallel forces acting as shown in fig below.



The forces acting are **W_1** acting downwards a distance **L_1** from the pivot, **W_2** acting downwards at a distance **L_2** from the pivot and the reaction, R, upwards at the pivot.

At equilibrium

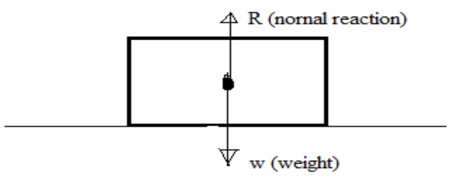
Condition1: $\sum F_y = R-(w_1+w_2)=0$

Condition 2: Take moments (torques) about O (Pivot)

$$\sum \tau = w_1L_1 - w_2L_2 = 0$$

3. Equilibrium of a body under the action of gravity:

(a)Equilibrium of a body on a horizontal plane



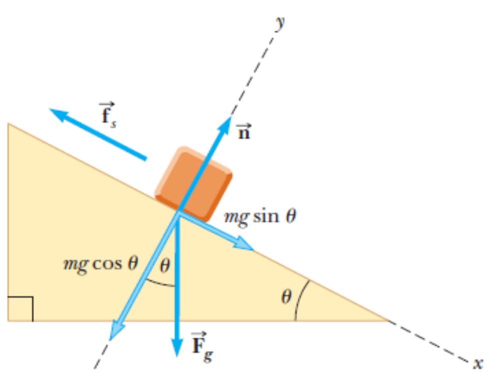
An object standing on a horizontal plane is in equilibrium because the plane exerts an upward reaction force that balances the object's weight.

Thus the total force is zero, according to Newton's third law, or from the condition of equilibrium

$\Sigma F_x = 0$, and $\Sigma F_y = 0$

$W - R = 0$, or $R = W = mg$

(b)Equilibrium of a body on an inclined plane



The force named above act on an object placed on an inclined plane:

i) The normal force (N) exerted on the body by the plane due to the attraction of the gravity $mgcos\theta$.

i) The force due to gravity acting parallel down to the plane $mgsin\theta$

ii) The force of friction f_s acting up parallel to the plane

The body is in equilibrium if and only if:

$\Sigma F_x = o$, and $\Sigma F_y = 0$

$\begin{cases} mgsin\theta - f_s = 0 \\ n - mgcos\theta = 0 \end{cases}$, or $\begin{cases} mgsin\theta = f_s \\ n = mgcos\theta \end{cases}$, $f_s = \mu_s n$

Therefore, $\frac{f_s}{n} = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta$

$\theta = \tan^{-1}\left(\frac{f_s}{n}\right)$, but $f_s = \mu_s n$

$\tan\theta = \frac{\mu_s n}{n} = \mu_s$, therefore, $\theta = arctan(\mu_s)$

Or $\theta = tan^{-1}(\mu_s)$.

Example: Example 1

A box of weight W rests on the sloping plank. The coefficient of static friction between the surfaces is 0.25. If the slope of the plank is gradually increased, at what angle of the slope will the box begin to slide?

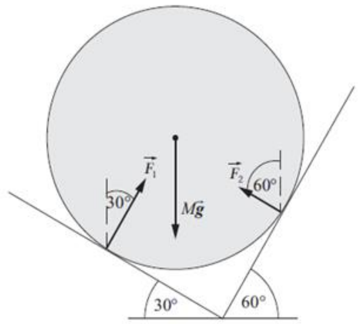
Solution

$\tan\theta = \mu_s = 0.25$

$\theta = tan^{-1}(0.25) = 14^0$

$\theta = 14^0$

The planes are frictionless; therefore, the force exerted by each plane must be perpendicular to that plane. Let $\vec{F}_1$ be the force exerted by the 30° plane, and let $\vec{F}_2$ be the force exerted by the 60° plane. Choose a coordinate system in which the positive x direction is to the right and the positive y direction is upward. Because the cylinder is in equilibrium, we can use the conditions for translational equilibrium to find the magnitudes of $\vec{F}_1$ and $\vec{F}_2$.



Solution

Apply $\sum F_x = 0$ to the cylinder: $F_1 \sin 30° - F_2 \sin 60° = 0$ (1)

Apply $\sum F_y = 0$ to the cylinder: $F_1 \cos 30° + F_2 \cos 60° - Mg = 0$ (2)

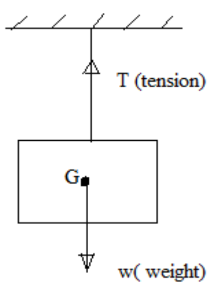
Solve equation (1) for F_1: $F_1 = \sqrt{3} F_2$ (3)

Substitute for F_1 in equation (2) to obtain: $\sqrt{3} F_2 \cos 30° + F_2 \cos 60° - Mg = 0$

Solve for F_2 to obtain: $F_2 = \frac{Mg}{\sqrt{3}\cos 30° + \cos 60°} = \boxed{\tfrac{1}{2} Mg}$

Substitute for F_2 in equation (3) to obtain: $F_1 = \sqrt{3}\left(\tfrac{1}{2} Mg\right) = \boxed{\tfrac{\sqrt{3}}{2} Mg}$

(c) Equilibrium of an object suspended



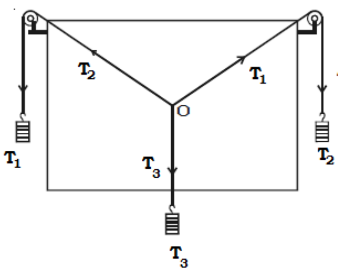
An object is in equilibrium if
$\Sigma F_x = 0$, and $\Sigma F_y = o$
$\Sigma F_y = w - T = o$, or $T = w$

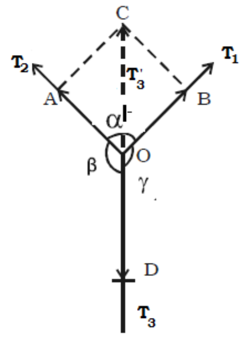
Two strings attached to a beam supporting a mass

Lami's theorem

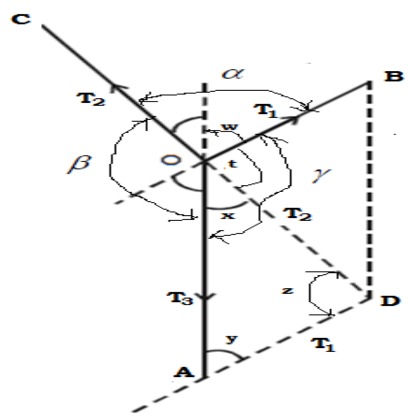
It gives the conditions of equilibrium for three forces acting at a point.

Lami's theorem states that *if three forces acting at a point are in equilibrium, then each of the force is directly proportional to the sine of the angle between the remaining two forces.*





Let forces T_1, T_2 and T_3 acting at a point O be in equilibrium.



Applying the sine rule to the triangle OAD gives:

$$\frac{T_1}{\sin x} = \frac{T_2}{\sin y} = \frac{T_3}{\sin y}$$

$$\frac{T_1}{\sin(180^0 - \beta)} = \frac{T_2}{\sin(180^0 - \gamma)} = \frac{T_3}{\sin(180^0 - \alpha)}$$

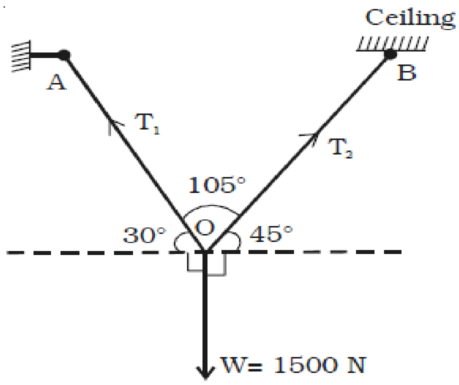
$$\frac{T_1}{\sin \beta} = \frac{T_2}{\sin \gamma} = \frac{T_3}{\sin \alpha}$$

Which is Lami's formula

Worked examples

Example1

A machine weighing 1500 N is supported by two chains attached to some point on the machine. One of these ropes goes to a nail in the wall and is inclined at 30° to the horizontal and other goes to the hook in ceiling and is inclined at 45° to the horizontal. Find the tensions in the two chains.



Solution

1st method

Now applying Lami's theorem at O, we get

$$\frac{T_1}{\sin(90^\circ+45^\circ)}=\frac{T_2}{\sin(90^\circ+30^\circ)}=\frac{T_3}{\sin 105^\circ}$$

$$\frac{T_1}{\sin 135^\circ}=\frac{T_2}{\sin 120^\circ}=\frac{1500}{\sin 105^\circ}$$

$$T_1=\frac{1500\times\sin 135^\circ}{\sin 105^\circ}=1098.96\,N$$

$$T_2=\frac{1500\times\sin 120^\circ}{\sin 105^\circ}=1346.11\,N$$

2nd method

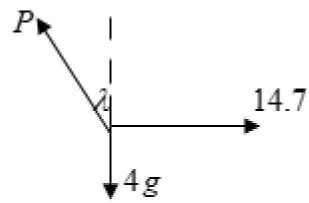
$\Sigma F_x = 0$, and $\Sigma F_y = 0$

$$\begin{cases}-T_1\cos 30^0+T_2\cos 45^0=0\\ T_1\sin 30^0+T_2\sin 45^0-w=0\end{cases}$$

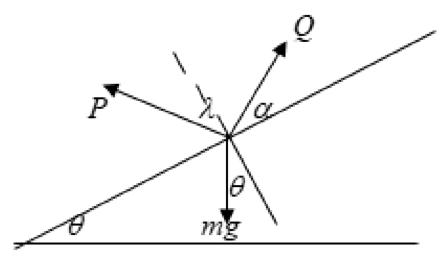
$$\begin{cases}-T_1\cos 30^0+T_2\cos 45^0=0\\ T_1\sin 30^0+T_2\sin 45^0-1500=0\end{cases}$$

$T_1=1098.96N$ and $T_2=1346.11N$

Example 2: A particle of mass $4kg$ is acted on by forces P ,14.7 in Newton as shown in the figure. If the particle is in limiting equilibrium and the angle of friction is λ , find the value of λ and the magnitude of force P .



Example 3:



$$\frac{P}{\sin(90^0+\theta+\alpha)}=\frac{Q}{\sin\left[180^0-(\lambda+\theta)\right]}=\frac{mg}{\sin(90^0-\alpha+\lambda)}$$

$$\frac{Q}{\sin(\lambda+\theta)}=\frac{mg}{\cos(\alpha-\lambda)}\text{ then}$$

$$Q=\frac{mg\sin(\lambda+\theta)}{\cos(\alpha-\lambda)}$$

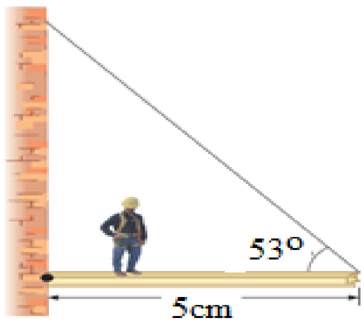
The expression for Q has a minimum value when $\cos(\alpha-\lambda)=1$ or when $\alpha=\lambda$

In this case $Q=mg\sin(\alpha-\lambda)$ and the force must be applied at an angle λ to the surface of the plane. The minimum value of Q is $mg\sin(\lambda+\theta)$ when the angle $\alpha=\lambda$.

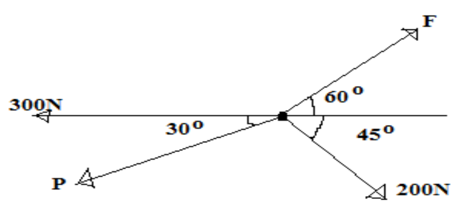
Example

A uniform horizontal beam 5.00 m long and weighing $3\times10^2 N$ is attached to a wall by a pin connection that allows the beam to rotate.

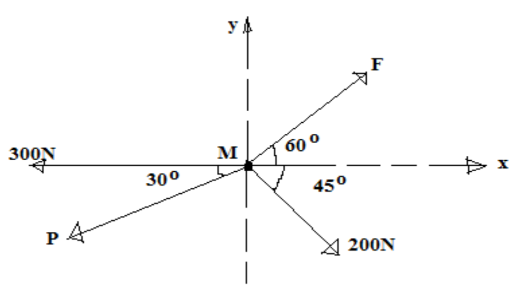
Its far end is supported by a cable that makes an angle of 53^0 with the horizontal. If a person weighing $6 \times 10^2 N$ stands 1.50 m from the wall, find the magnitude of the tension in the cable and the force $\vec{R}$ exerted by the wall on the beam.



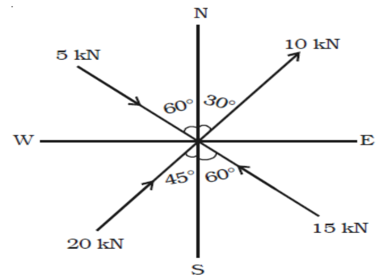
Example4



In static, a body is said to be in equilibrium when the force system acting upon it has a zero resultant.
To determine the values of P and F, when the body M is in equilibrium, use the projection of the force on the x and y-axis.



Activity
Determine analytically the magnitude and direction of the resultant of the following four forces acting at a point.
i.10 kN pull N 30° E; (ii) 20 kN push S 45° W;
ii. 20 kN push S 45° W;
iii5 kN push N 60° W; (iv) 15 kN push S 60° E.
iv. 15 kN push S 60° E.

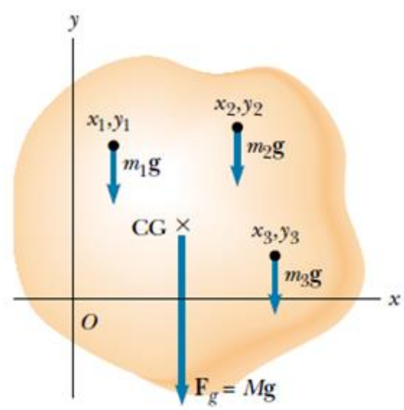


Ans: 29.7kN and the direction is at an angle of 69.34^0

4. Centre of gravity and the total weight

1. Center of gravity

Centre of gravity (C.G) is defined as a fixed point in the body where the weight of the body acts.
To find the centre of gravity, X_{CG} , consider an object of arbitrary shape lying in the xy plane, as illustrated in Figure below.



total mass of the object M = $m_1 + m_2 + m_3 + ...$
Equating the torque resulting from **Mg** acting at the center of gravity to the sum of the torques acting on the individual particles gives

$$(m_1g_1 + m_2g_2 + m_3g_3 + ...)x_{CG} = m_1g_1x_1 + m_2g_2x_2 + m_3g_3x_3 + ...$$

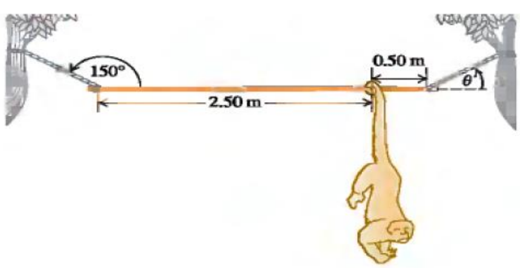
$$x_{CG} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + ...}{m_1 + m_2 + m_3 + ...}$$

Exercises

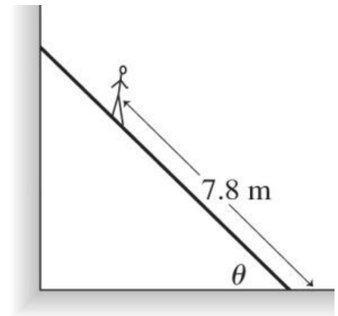
1. A 65 kg student exerts a force of 55 N on the end of a 25 kg door 74 cm wide. What is the magnitude of the torque if the force is exerted (*a*) perpendicular to the door, and (*b*) at a 30º angle to the face of the door?

2. A 5.00 m long ladder leans against a frictionless wall. The point of constant between the ladder and the wall is 4.00 m above the ground. The ladder is uniform with a mass of 15.00 kg. Determine the forces exerted by the ground and the wall on the ladder

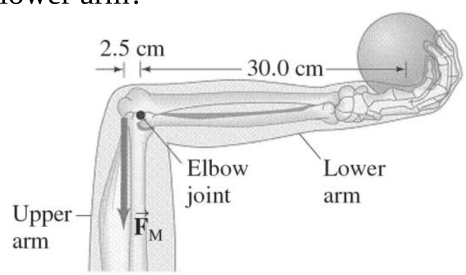
3. A 3.00m long, 240N, uniform rod at zoo is held in a horizontal position by two ropes at its end as shown below. The left rope makes an angle of 150^0 with the rod and the right rope makes an angle θ with the horizontal. A 90N bowler monkey *(Alouatta seniculus)* hangs motionless 0.50m from the right end of the rod as he carefully studies you. Calculate the tension in two ropes and the angle θ. Fist make a free-body diagram of the rope ./ **7marks**



4. A uniform ladder 12 meters long rests against a vertical frictionless wall, as shown in the figure. The ladder weighs 400 N and makes an angle *θ* = 51° with the floor. A man weighing 874 N climbs slowly up the ladder. When he is 78 m from the bottom of the ladder, it just starts to slip. What is the coefficient of static friction between the floor and the ladder?

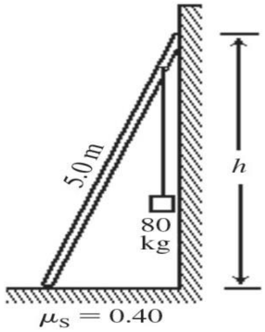


5. An athlete holds a 7.5-kg shot put in his hand with his lower arm horizontal, as shown in the figure. His lower arm has a mass of 2.8 kg and its center of gravity (or center of mass) is 12 cm from the elbow-joint pivot. How much force must the extensor muscle (which is $\vec{F}_M$ in the figure) in the upper arm exert on the lower arm?



6. A 3.00-m-long ladder, weighing 200 N, leans against a smooth vertical wall with its base on a horizontal rough floor, a distance of 1.00 m away from the wall. The ladder is not completely uniform, so its center of gravity is 1.20 m from its base. What force of friction must the floor exert on the base of the ladder to prevent the ladder from sliding down?

7. A 40-kg uniform ladder that is 5.0 m long is placed against a smooth wall at a height of *h* = 4.0 m, as shown in the figure. The base of the ladder rests on a rough horizontal surface whose coefficient of static friction with the ladder is 0.40. An 80-kg bucket is suspended from the top rung of the ladder, just at the wall. What is the magnitude of the force that the ladder exerts on the wall?

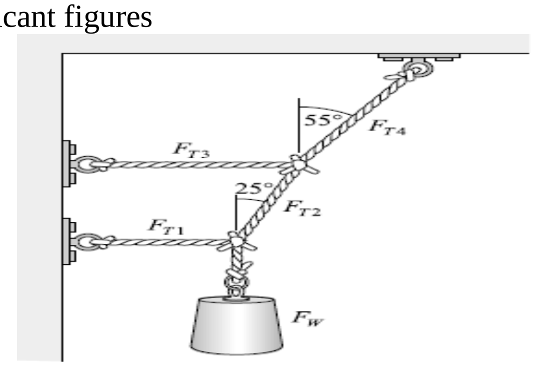


8. A 350 N, uniform, 1.5 m bar is suspended horizontally by two vertical cables at each end. Cable A can support a maximum tension of 500 N without breaking, and cable B can support up to 400 N. You want to place a small weight on this bar.

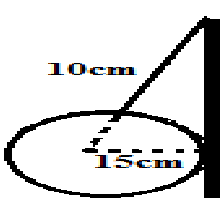
a) What is the heaviest weight that can put on without breaking either cable?

b) Where this weight should be put?

9. The object in the figure below is in equilibrium and has a weight $F_w = 80N$. Find F_{T1}, F_{T2}, F_{T3} and F_{T4}. Give answers to two significant figures



10. A sphere of weight 20N and radius 15cm rests against a smooth vertically wall. The sphere is supported in this position by a string of length 10cm attached to a point on the sphere and to a point on the wall as shown below



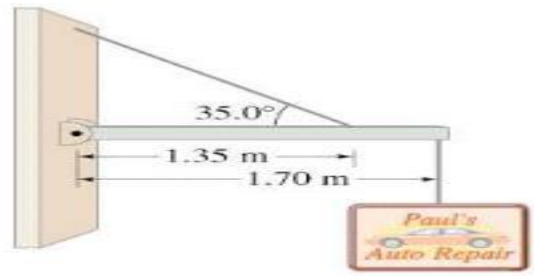
a) Copy the diagram and show the forces acting on the sphere

b) Calculate the reaction on the sphere due to the wall

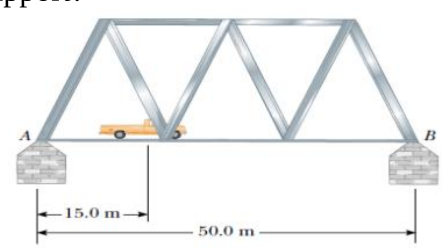
c) Find the tension in the string.

d) Find the resultant force on the object

11. A shop sign weighing 245N is supported by a uniform 155N beam as shown in figure below. Find the tension in the guy wire and the horizontal and vertical forces exerted by the hinge on the beam



12. A bridge of length 50.0 m and mass 8.00 x10^4kg is supported on a smooth pier at each end as in Figure below. A truck of mass 3.00 x10^4 kg is located 15.0 m from one end. What are the forces on the bridge at the points of support?



13. A uniform beam of length L weights 200 N and holds a 450 N object as shown in Figure below. Find the magnitudes of the forces exerted on the beam by the two supports at its ends. Assume the lengths are exact.