UNIT 1: WAVES AND PARTICLE NATURE OF LIGHT

1.0. Introduction

In physics, a wave is an oscillation accompanied by a transfer of energy that travels through a medium (space or mass).Wave motion transfers energy from one point to another, which displace particles of transmission medium that is, with little or no associated mass transport. There are two main types of waves: Mechanical waves propagate through a medium, and the substance of this medium is deformed. The second main type, electromagnetic waves, do not require a medium.

1.1. Planck's quantum theory

Quantum theory consists of facts concerning the interactions of electromagnetic radiation with matter. Two of these interactions are photoelectric emission and black body radiation. Both light and matter consists of tiny particles which have wavelike properties associated with them. **Light is composed of photons** and **matter is composed of electrons, protons and neutrons**.

The quantum theory arose out of the inability of classical physics to explain the experimentally observed distribution of energy in the spectrum of a black body. When a black body is heated, it emits thermal radiations of different wavelengths or frequencies. To explain these radiations, Max Planck put forward a theory known as Planck's quantum theory which is summarized in the following statements:

1. The matter is composed of a large number of oscillating particles, having different frequencies.
2. The radiant energy which is emitted or absorbed by the black body is not continuous but discontinuous in the form of small discrete packets of energy and each such packet of energy is called a "quantum". In the case of light, the quantum of energy is called a "photon".
3. The energy of each quantum is directly proportional to the frequency of the radiation i.e. $E \propto f$ or $E = hf$ then $f = \frac{c}{\lambda}$, $E = \frac{hc}{\lambda}$ where c is the speed of light, λ is the wavelength and h is the Planck's constant.
4. The oscillator emits energy, when it moves from one quantized state to the other quantized state. The oscillator does not emit energy as long as it remains in one energy state. The total amount of energy emitted or absorbed by a body will be some whole number quanta. Hence $E = nhf = \frac{nhc}{\lambda}$ where n is an integer.

According to the Planck's quantum theory, the exchange of energy between quantized states is not continuous but discrete. This quantized energy is in small packets of bundles.

Max Plank supposed that "***A body would thus emit one, two, three etc quanta of energy but no fractional amount"s.***

According to Plank, the quantum E of energy for radiation of frequency f is given by $E = hf$ where h is Plank's constant. Its value is $h = 6.63 \times 10^{-34}\ Js$. For electromagnetic radiation of wavelength λ, the speed $c = f\lambda$, where c is its speed in vacuum and so we have $E = h\frac{c}{\lambda}$. The speed of electromagnetic wave is given by $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \frac{E}{B}$ where μ_0, ε_0 are the permeability and permittivity of free space and

$(\mu_0 = 4\pi \times 10^{-7} Hm^{-1}$ and $\varepsilon_0 = 8.854187817620... \times 10^{-12} Fm^{-1})$ or $\varepsilon_0 = \frac{1}{\mu_0 c^2}$. The energy of a quantum is thus inversely proportional to the wavelength of the radiation but directly proportional to the frequency. It is convenient to express many quantum energies in electro-volts. The quantum for red light has energy of about $2eV$ and for blue light; it is about $4eV$ by using the equation $E = hf$.

1.2. Photon theory of light and photoelectric effect

As proposed by Einstein, light is composed of photons, very small packets of energy. A photon is an elementary particle, the quantum of all forms of electromagnetic radiation including light, whose rest mass equals zero (i.e. $m_{photon} = 0$) and has no electric charge ($q_{photon} = 0$).

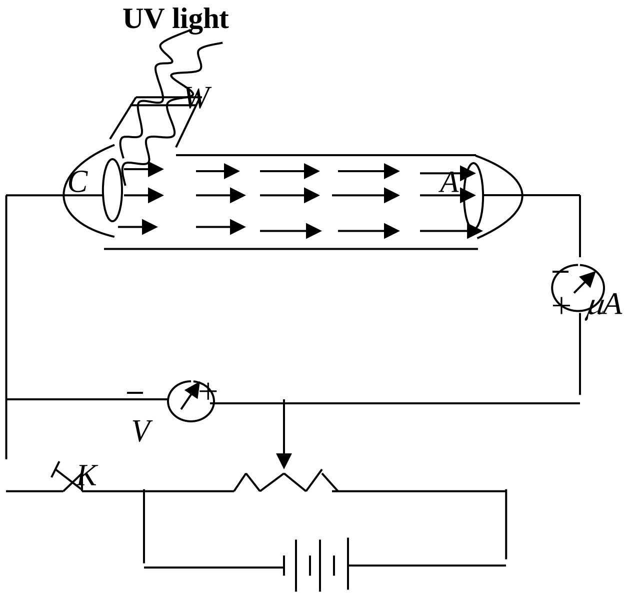
1.2.1. Properties of photons

- A photon travels at a speed of light c in vacuum (i.e. $3 \times 10^8 ms^{-1}$)
- It has zero rest mass (it has no mass) i.e. the photon cannot exist at rest.
- It has zero charge (it has no charge)
- The kinetic mass of a photon is, $m = \frac{E}{c^2} = \frac{h}{\lambda c}$
- The momentum of a photon is, $p = \frac{E}{c} = \frac{h}{\lambda}$.
- Photons travel in a straight line.
- It is a carrier of electromagnetic energy. Energy of a photon depends upon frequency of the photon; so the energy of the photon does not change when photon travels from one medium to another

1.2.2. Photoelectric effect

2.0. Photoelectric effect is the emission of electrons from the surface of metals when light of a certain frequency is incident on it. In other words, it is the process of removal of electrons from the surface of metal when the rays of special frequency fall on the surface of metal. As a result of the flow of these photoelectrons, **the photoelectric current is produced**.

Experimental Set-up to study Photoelectric Effect:



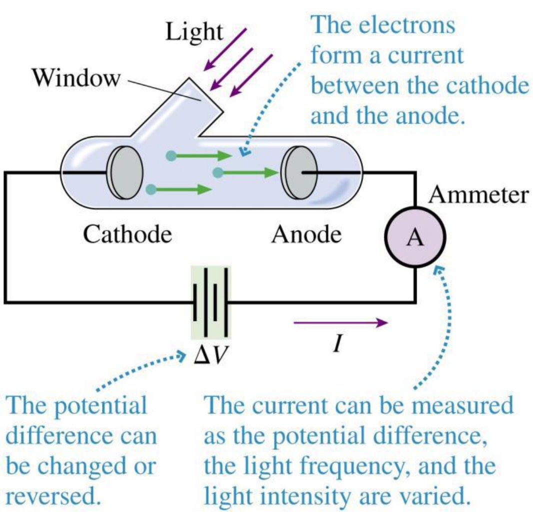
C : Metallic Cathode

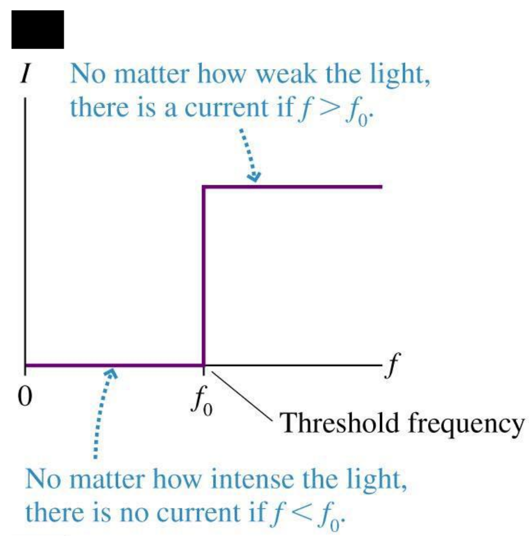
A : Metallic Anode

W : Quartz Window

: Photoelectron

Glass transmits only visible and infra-red lights but not UV light. Quartz transmits UV light.





The electron that gets kicked out of the metal gets its energy from that photon. Some of the energy is used to break the electron from the metal (**the work function** Φ is the amount of energy binding the electron to the metal) that's. $KE_{max} = hf - \Phi$

There is a critical frequency for each metal, ν_0, below which no electrons are emitted; this is called ***threshold frequency***.

Now we can write an equation for the kinetic energy of the emitted electron. $KE = hf - hf_0$ where KE is the kinetic energy of electron emitted from metal, hf is the energy of photon and hf_0 is the energy needed to eject an electron from the metal; called the ***work function of a metal***.

The intensity of radiation is the power per unit area $I = \frac{P}{A} = \frac{nhf}{tA}$ or $I = \frac{nh\frac{c}{\lambda}}{tA} = \frac{nhc}{\lambda tA}$ and the rate at which electros are emitted is $\frac{n}{t} = \frac{A}{nf}I = \frac{P\lambda}{hc}$ where P is the power.

The photocurrent produced by n electrons is $i = \frac{Q}{t} = \frac{ne}{t} = \frac{AIe}{hf} = \frac{Pe}{h\frac{c}{\lambda}} = \frac{Pe\lambda}{hc}$.

2.1.1. Applications of photoelectric effect

There are numerous desirable applications based on photoelectric effect such as:

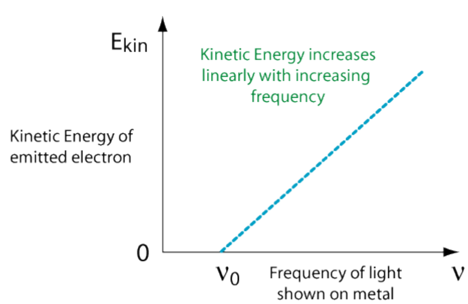
1. Automatic doors: the beam of light strikes the photocell, the photoelectric effect generates enough ejected electrons to produce a detectable electric current. When the light beam is blocked (by a person), the electric current is interrupted and the doors are signalled to open.
2. Solar panels: photocells convert sunlight into electrical energy.
3. Automatic fire alarm
4. Automatic burglar alarm
5. Scanners in Television transmission
6. Reproduction of sound in cinema film
7. In paper industry to measure the thickness of paper
8. To locate flaws or holes in the finished goods
9. Automatic switching of street lights
10. Photometry

2.1.2. Measure of Planck's constant

Planck's constant connects the particulate photon energy E with the associated wave frequency *f*.

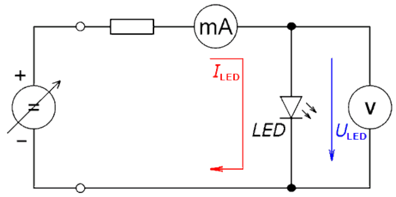
The electron is emitted from the metal with a specific kinetic energy (*i.e.* a specific speed). The kinetic energy of the emitted electron must depend on the frequency of the light and this changed the kinetic energy of the emitted electron.

When the kinetic energy of emitted electron is plotted against the frequency, the slope of the line represents Planck's constant h. We call a reduced Planck's constant the symbol $\hbar = \frac{h}{2\pi}$.



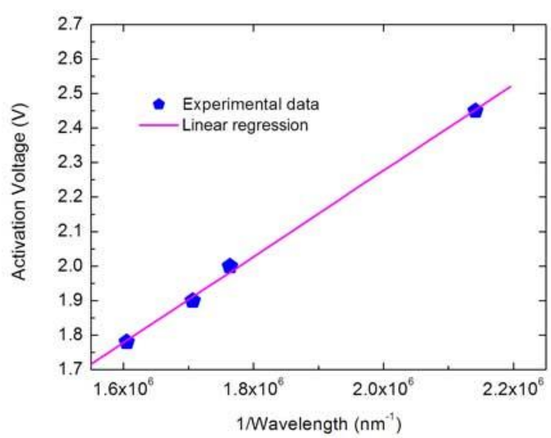
The determination of this constant requires the following apparatus:

- $0-10V$ power supply,
- a digital milliameter,
- a digital voltmeter,
- a $1k\Omega$ resistor and
- Different known wavelength LEDs (Light Emitting Diodes).



A LED (Light Emitting Diode) is a tow terminal semiconductor light source. The light energy emitted by LED is given as $E=\frac{hc}{\lambda}$ (1). If V is the forward voltage applied across the LED when it begins to emit light, the energy given to electrons crossing the junction is eV (2). Equating both equations, we get $eV=\frac{hc}{\lambda}$ (3) and $h=\frac{e}{c}\lambda V$. The voltage V can be measured for LEDs with different values of λ (wavelengths of lights) $V=\frac{hc}{e}\times\frac{1}{\lambda}$ (4).

Now from the equation (4), we see that the slope S of the graph of V on the vertical axis and $\frac{1}{\lambda}$ on the horizontal axis is $S=\frac{hc}{e}$ (5).



To determine Planck's constant h, we take the slope S from our graph and calculate $h=S\frac{e}{c}$. Using known values $\frac{e}{c}=5.33\times10^{-28}\,Csm^{-1}$. Alternatively, we can write equation (3) as $h=\frac{e}{c}\lambda V$, calculate h for each *LED*, and take the average of our results.

1.3. Wave theory of monochromatic light

1.3.1. Definitions

A wave is any disturbance that results into the transfer of energy from one point to another point.

Primary source: the geometrical centre or axis of the actual source of light which is either a point or a line is called the primary source.

Wavelets: all points lying on small curved surfaces that receive light at the same time from the source (primary or secondary) are called wavelets.

Secondary source: any point on a wavelet, acts as the source of light for further propagation of light. It is called a secondary source.

Wave front: This is the envelope of all wavelets in the same phase receives light from sources in the same phase at the same time.

Wave normal: This is the normal at any point drawn outward on a wave front. Further propagation of light occurs along the wave normal. In isotropic media, the wave normal coincides with the 'ray of light'.

1.3.2. Huygens 'principle of monochromatic light

Huygens published a theory in 1690, having compared the behaviour of light not with that of water waves but with that of sound. According to Huygens' Principle:

- Light travels in the form of longitudinal waves which travel with uniform velocity in homogeneous medium
- Different colours are due to the different wavelengths of light waves.
- We get the sensation of light when these waves enter our eyes.
- In order to explain the propagation of waves of light through vacuum. Huygens suggested the existence of a hypothetical medium called aluminiferous ether, which is present in vacuum as well as in all material objects. Since ether couldn't be detected, it was attributed properties like:
  - It is continuous and is made up of elastic particles
  - It has zero density
  - It is perfectly transparent
  - It is present everywhere.

The Huygens 'principle of the wave theory of light states that: ***"Every point on the wave front may be considered as a source of secondary spherical wavelets which spread out in the forward direction at the speed of light. The new wave front is the tangential surface to all of these secondary wavelets".***

In the wave theory assuming monochromatic light, the two important properties of light wave are its intensity and frequency (or wavelength). When these two quantities are varied, the wave theory makes the following predictions:

1. If the light intensity is increased, the number of electrons ejected and their maximum kinetic energy should be increased because the higher intensity means greater electric field amplitude, and the greater electric field should eject the electrons with higher speed.
2. The frequency of the light should not affect the kinetic energy of the ejected electrons. Only the frequency of incident radiation affect the maximum kinetic energy. there are however, at least two problems with this idea and these led Newton and others to reject it:
   - The secondary waves are propagated in the forward direction only and
   - They are assumed to destroy each other except when they form the new wave front.

1.3.3. Limitations of Huygens 'wave theory of light

It could not explain the rectilinear propagation of light

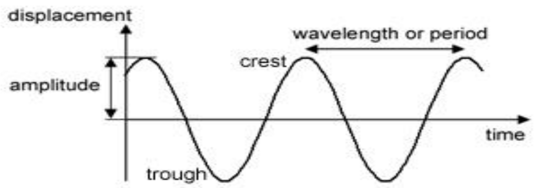
It could not explain the phenomena of polarization of light such as Compton and photoelectric effect

Michelson and Morley experiment concluded that there is no ether drag when the earth moves through it. This proves ether doesn't exist. All other attempts/experiments to detect aluminiferous ether failed, which prove that it does not exist.

1.4. Representation and Properties of light wave

Representation of wave

Wave can be represented on a ***distance*** or ***time*** graphs. The graph shows how the displacement of particles varies along a wave. The displacement and time have coefficients k and ω respectively.



Characteristics of wave

1. **Wave number** k is the number of waves in a unit distance this is the spatial frequency of a wave either in cycles per unit distance or radians per unit distance. Mathematically $k = \frac{2\pi}{\lambda}$; it is also called **the propagation constant of wave motion.**
2. **Crest** is the top, maximum height of the transverse wave.
3. **Trough** is the bottom or lowest point of the transverse wave.
4. **Displacement** x is the distance a particle moves from its central equilibrium position.
5. **Amplitude** x_0 is the maximum displacement from the central equilibrium
6. **Time period** T . This is the time it takes for the wave to travel a complete wavelength. Mathematically $T = \frac{2\pi}{coefficient\ of\ t}$ i.e. $T = \frac{2\pi}{\omega}$
7. **Frequency** f is the number of complete waves (cycles) per unit time; mathematically $f = \frac{coefficient\ of\ t}{2\pi}$ i.e. $f = \frac{\omega}{2\pi}$.
8. **Wavelength** λ is a displacement equivalent to one complete wave; this is the shortest distance along the wave between two points that are in phase with one another i.e. the distance over which the wave's shape repeats. Mathematically the wavelength is $\lambda = \frac{2\pi}{coefficient\ of\ x}$ i.e. $\lambda = \frac{2\pi}{k}$.

   Phase angle is the angle in degrees or radians that the waveform has shifted from a certain reference point along the horizontal zero axis. . One complete wave is 360^0 or 2π radians; so from a peak to the through will be a change in phase of 180^0 or π radians. It is denoted by $\phi = kx$ so that $\phi = 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$.

To calculate the phase angle, one should proceed as follows: $\lambda \to 2\pi$, $1 \to \frac{2\pi}{\lambda}$, $x \to \frac{2\pi x}{\lambda}$.

9. **Speed of wave** v or wave velocity is the speed at which the wave fronts pass a stationary object. $v = \frac{coefficient\ of\ t}{coefficient\ of\ x}$ i.e. $v = \frac{\omega}{k}$ where $k = \frac{2\pi}{\lambda}$ or $v = \frac{\omega}{\frac{2\pi}{\lambda}} = \frac{\omega\lambda}{2\pi}$. If the time for one complete wave is the time period T and the distance is the wavelength λ, **then**

$speed = \frac{distance}{time}$; $v = \frac{\lambda}{T}$. But as the frequency $f = \frac{1}{T}$, $v = \frac{\lambda}{T}$ becomes $v = \lambda f$.

10. **Intensity** of the wave is the power per unit area that is received by a stationary observer. The intensity is directly proportional to the square of the amplitude. $I = \frac{P}{A}$ and $I \propto x_0^2$.

PROPERTIES OF LIGHT WAVE

The wave has the following properties: Reflection, Refraction, Diffraction, Interference and Polarization.

1.5. Blackbody radiation

A blackbody is a theoretical object that absorbs 100% of the radiation that hits it and re-radiates energy which is the characteristic of this radiating system or body only. ***It is a hypothetical perfect absorber and radiator of energy with no reflecting power***; therefore it reflects no radiation and appears perfectly black.

A blackbody is a surface that

• Completely absorbs all incident radiation

• emits radiation at the maximum possible monochromatic intensity in all directions and at all wavelengths.

The blackbody radiation is the emission of electromagnetic waves from the surface of an object. The distribution of blackbody radiation depends on the temperature of the object and is independent of the material.

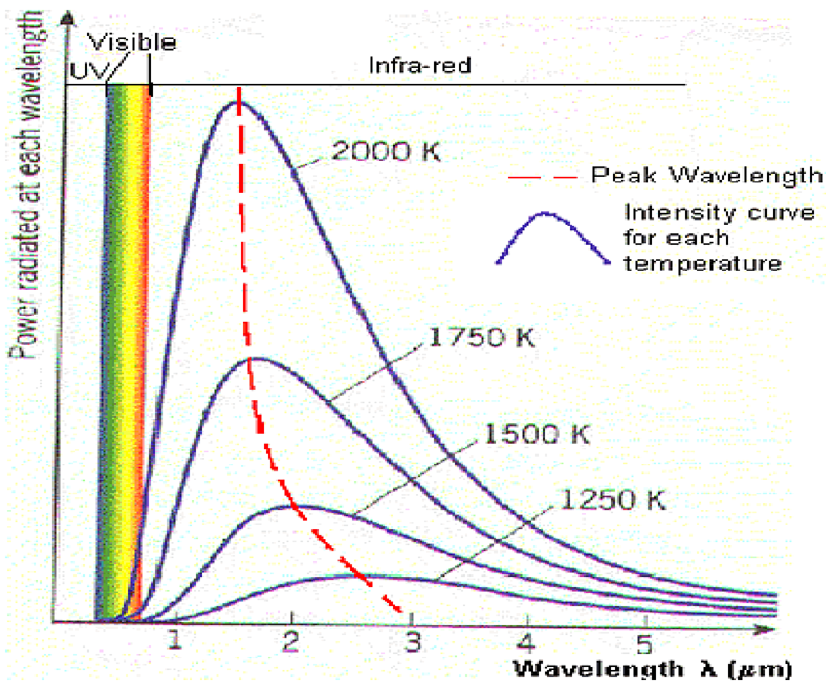
We would expect a black body to be the best possible emitter at any given temperature. The radiation emitted by it is called ***"black body radiation"*** or ***"full radiation" or "temperature radiation".*** The energy in the spectrum of a black body is distributed among the various wavelengths. As in the diagram below

1.5.1. Wien's displacement law

Wien's law states that ***"the wavelength of peak emission is inversely proportional to the temperature of the emitting object"*** i.e. the hotter the object, the shorter the wavelength of maximum emission.

i) As the temperature rises, the energy emitted in each band of wavelength increases, the body becomes "brighter".

ii) The statement is known as ***Wien's displacement law***. This explains why a body appears successively red-hot, yellow-hot and white-hot etc. Sirius (Dog Star) looks blue not white. The full equation is $\lambda_{\max} = \frac{0.29 \times 10^{-2}}{T} m$.



1.5.2. Stephan's law of radiation

This states that ***"the total energy*** E ***radiated of all wavelengths per unit area per unit time by a black body is directly proportional to the fourth power of the thermodynamic temperature*** T ***".*** Mathematically, $E \propto T^4$ i.e. $E = \sigma T^4$ where σ is the Stephan's constant. Its value is $5.7 \times 10^{-8} Wm^2 K^{-4}$.

1.5.3. Kirchhoff's law of radiation

It states that ***"for an object whose temperature is not changing, an object that absorbs radiation well at a particular wavelength will also emit radiation well at that wavelength".***

1.6. Energy, mass and momentum of a photon

The energy-momentum relation is the relativistic equation relating any object's rest mass, total energy, and momentum:

$$E^2 = (pc)^2 + (m_0 c^2)^2$$

The famous Einstein's equation of energy of the photon is $E = mc^2$. In short, the equation describes how energy and mass are related with speed of light. To derive this equation, consider X-ray photon of mass m hitting the surface of a metal and consider if a part of its energy is gained by a surface electron and is then emitted.

The most important laws in dynamics are those that state the conservation of energy and the conservation of momentum. Those two laws can be applied whenever we have a closed system; that is, a system that does not interact with its surroundings. They assert that for such systems and any process they may undergo:

assume that E is the energy, s is the distance, F is the force, c is the speed of the photon, t is the time and p is the momentum, then $\sum E_i = \sum E_f$ and $\sum p_i = \sum p_f$.

The total energy of the photon is given by $E = Fs$

The distance moved by the photon is $s = ct$, thus $E = Fct$

From Newton's second law, $F = \frac{p}{t}$ (force equals the rate of change of momentum); the momentum of the photon is given by $p = Ft = mc$; and we deduce that $F = \frac{mc}{t}$.

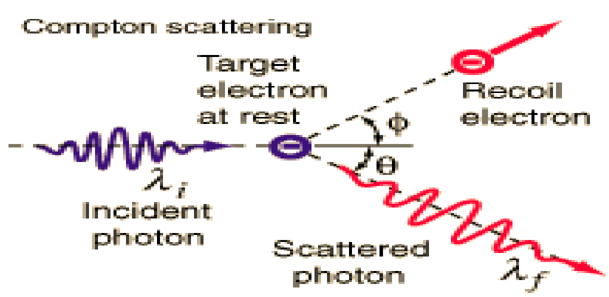
Substituting for s in the equation for energy, we get the total energy of the photon given by $E = \frac{mc}{t}ct = mc^2$. The equation for energy can be written as $E = \frac{p}{t}ct = mc^2$ from which we get $E = pc = mc^2$ and we deduce $p = \frac{E}{c}$.

THE PARTICLE NATURE OF LIGHT

Compton effect

Compton in 1923, his experiment provides additional confirmation of the quantum nature of $X-rays$. He discovered that when X-rays strike or fall on matter (metal) some of radiations are scattered and the scattered radiation has small frequency (longer wavelength) than the incident radiation and the change in wavelength depends on the angle through which the radiation is scattered. This was known as ***"Compton effect''*** or ***"Compton scattering".***

The Compton Effect concerns the inelastic scattering of X-rays by electrons as illustrated below



Before collision After collision

Compton Effect is the result of a high energy photon colliding with a target which releases loosely bound electrons from the outer shell of the atom or molecule. Some of the energy and momentum of the photon is transferred to the electron. The scattering radiation experiences a wavelength shift that cannot be explained in terms of classical wave theory, thus lending support to Einstein's photon theory.

The Compton shift $\Delta\lambda = \lambda' - \lambda$ is the change in wavelength due to the loss of energy of the incident X-rays. If the scattered radiation energies at an angle θ with respect to the incident direction and if λ and λ' are the wavelengths of incident and scattered radiations, respectively, he found that

Energy of a photon before collision $E = hf = \frac{hc}{\lambda}$

Energy of a photon after collision $E' = hf' = \frac{hc}{\lambda'}$

Conservation of energy $\frac{hc}{\lambda} = \frac{hc}{\lambda'} + K.E.$

Combining with conservation of momentum $\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\theta)$ where m_0 is electron rest mass, h is Planck's constant and c is the speed of light.

Note that, the classical theory (wave theory) was not able to explain Compton Effect because it predicts that scattered wave has the same wavelength as the incident wave. But the quantum theory provides a beautifully clear explanation. They imagine the scattering process as collision of two particles: the incident photon and electron that is initially at rest.
We can also say that the incident photon transfers a part of its energy to the free electron (target electron) and the scattered photon must have a lower energy. From the equation $\lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\theta)$, when

$\theta = 0^0$, $\Delta\lambda = 0$

$\theta = 90^0$, $\Delta\lambda = \frac{h}{m_0 c} = 2.426 \times 10^{-12} m$. The value $\frac{h}{m_e c}$ is called ***Compton wavelength*** of the electron and has a value of $\Delta\lambda \approx 0.00243 nm$ and θ is the scattering angle of photon. This is not of course an actual wavelength, but really a ***proportionality constant for the wavelength shift.*** .

1.7. Photon interactions

A photon can be considered to have a wavelength and frequency (like a wave), as well as momentum and energy (like a particle).

Photons are electromagnetic radiations with zero mass, zero charge and a velocity that is always equal to the speed of light. A photon has no 'charge' and has a much lower chance of interacting with matter than charged particles such as electrons and protons. Photons travel some considerable distance before undergoing a more "**catastrophic" interaction leading to partial or total transfer of the photon energy to electron energy**. These electrons will ultimately deposit their energy in the medium. Photons are far more penetrating than charged particles of similar energy.

1.7.1. Types of photon interaction

There are six ways in which photons may interact with matter and some photon interactions are important in therapy and/or diagnostic radiology. These may cause the photon to attenuate (lose some of its energy and/or disappear). Photon interactions are very important when considering how a photon beam interacts with a patient.

- **Incoherent Scattering, also known as Compton Scattering or Compton Effect**
- **Photoelectric Effect** * **Coherent Scattering**,
- **Pair Production** ***Triplet Production**
- **Photodisintegration**

a. **Incoherent Scattering (Compton Effect) - σ_{inc}**

Incoherent scattering is the **most important** interaction in radiotherapy. It occurs when a photon has a much greater amount of energy than the binding energy of the electron, effectively considering the electron as 'free'. In this interaction, the photon interacts with the 'free' electron, giving up some of its energy and undergoing scattering. The electron receives the energy and is set in motion in a different direction.

b. **Photoelectric Effect – τ :**

The photoelectric effect occurs when a photon interacts with an orbital electron whose binding energy is close to that of the photon energy. In this scenario, the photon disappears and all of its energy is given to the orbital electron, which is then ejected from the atom with kinetic energy.

c. **Coherent Scattering - σ_{coh}**

Coherent scattering occurs at low photon energy radiation. A photon may interact with an orbital electron and is then deflected (or scattered) at a small angle. It occurs when the energy of X-ray or gamma photon is small in relation to the ionization energy of the atom. Hence no emission of electrons

d. **Pair Production – κ**

Pair production is the creation of an elementary particle and its antiparticle i.e. a pair of particles whose charges are opposite, for example an electron (-1) and positron $(+1)$, a muon and antimuon, or a proton and antiproton. Example $Y \rightarrow e^- + e^+$.

It occurs when a photon passes very close to the nucleus of an atom. The photon interacts with the strong nuclear field of the atom, in such a way the photon transforms itself into an elementary particle-antiparticle pair. If the energy of the photon is high enough, the photon may disappear and 'create' an ***electron*** and ***a positron***

e. **Triplet Production - κ_{tr}**

It is possible for pair production to occur in proximity to an electron; this is called triplet production. Triplet production is a special case of pair production which occurs in the vicinity of an orbital electron. The photon disappears and the energy is used to create an electron and positron.

f. **Photodisintegration - π**

Photodisintegration is an uncommon event that occurs when a photon (high energy gamma ray) is absorbed by the nucleus of an atomic nucleus and causes it to enter an excited state which immediately decays by emitting subatomic particles such as a proton, neutron, or alpha particle.

1.8. Dual nature of radiation and matter (Wave particle duality of light and matter).

Wave particle duality is the concept that every elementary particle may be partly described in terms not only of particles but also of waves. It expresses the inability of the classical concepts "particle" or "wave" to fully describe the behavior of quantum-scale objects. A given kind of quantum object will exhibit sometimes wave, sometimes particle character; in respectively different physical settings.
Wave theory of electromagnetic radiations explained the phenomenon of **interference**, **diffraction** and **polarization**. On the other hand, **quantum theory** of electromagnetic radiations successfully explained the **Photoelectric Effect, Compton Effect, Black Body Radiations, X- ray Spectra**, etc.

Note: In **now experiment, matter exists both as a particle and as a wave simultaneously**.

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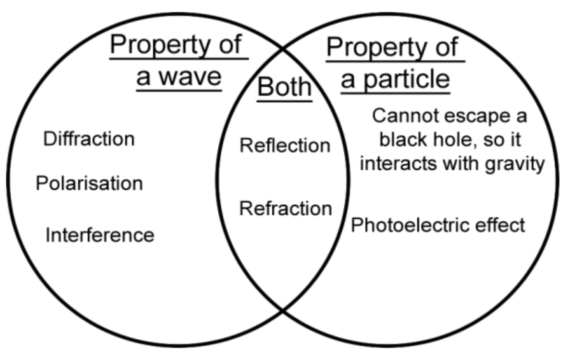
Wave-like Behavior of Light
Diffraction, ***interference*** and ***polarization*** are explained by the wave-like behavior of light.

The frequency of light is related to its wavelength according to the equation $f = \frac{c}{\lambda}$. $E = hf = h\frac{c}{\lambda}$ where h is Planck's constant.

Particle-like behavior of light.
Compton's effect and photoelectric effect support the particle-like behavior of light where it interacts with matter. Isaac Newton developed the ***corpuscular hypothesis*** when explaining rectilinear propagation of light.

The properties of light as wave-particle are summarized in the following diagram:



1.9. The principle of complementarities
The principle of complementarity refers to the effects such as wave particle duality in which different measurements made on the system reveal it to be complementary to each other; but at the same time, they also exclude each other.
Niels Bohr saw the duality as one aspect of the ***concept of complementarity.***

Examples of complementarity properties:

- Position and momentum, *Energy and duration,
- Spin on different axes, *Wave and particle,
- Value of a field and its change (at certain position),
- Entanglement and coherence.

De Broglie wave

According to de Broglie, a moving material particle can be associated with a wave. i.e. a wave can guide the motion of the particle. The waves associated with the moving material particles are known as de Broglie waves or matter waves.

Expression for de Broglie wave

According to quantum theory, the energy of the photon is $E = hf = \frac{hc}{\lambda}$

According to Einstein's theory, the energy of the photon is $E = mc^2$

So, $\lambda = \frac{h}{mc}$ or $\lambda = \frac{h}{p}$ where $p = mc$ is momentum of a photon

If instead of a photon, we have a material particle of mass m moving with velocity v, then the equation becomes $\lambda = \frac{h}{mv}$ which is the expression for de Broglie wavelength.

Different forms of de Broglie wavelength:

i) Relating wavelength and momentum $\lambda = \frac{h}{p} = \frac{h}{mv}$

ii) Relating wavelength and kinetic energy, $\lambda = \frac{h}{\sqrt{2mE_k}}$ since $p = \sqrt{2mE_k}$ for non-relativistic case.

iii) The kinetic energy of a charged particle carrying q charges is given by $E_k = qV$ where V is the accelerating potential then, $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mqV}}$.

1.10. Electron microscope

A microscope can be defined as an instrument that uses one or several lenses to form an enlarged (magnified) image. Microscopes can be classified according to the type of electromagnetic wave employed and whether this wave is transmitted or not through the specimen.

An **electron microscope** is a type of microscope that uses electrons to illuminate a specimen and create an enlarged image. It is an impressively powerful microscope that exists today, allowing researchers to view a specimen at nanometre size. Electron microscopes have much greater resolving power than light microscopes and can obtain much higher magnifications.

The electron microscope uses electrostatic lenses in forming the image by controlling the electron beam to focus it at a specific plane relative to the specimen in a manner similar to how a light microscope uses glass lenses to focus light on or through a specimen to form an image.

The most common electron microscopes are **Transmission Electron Microscope** (TEM) and **Scanning Electron Microscope** (SEM).

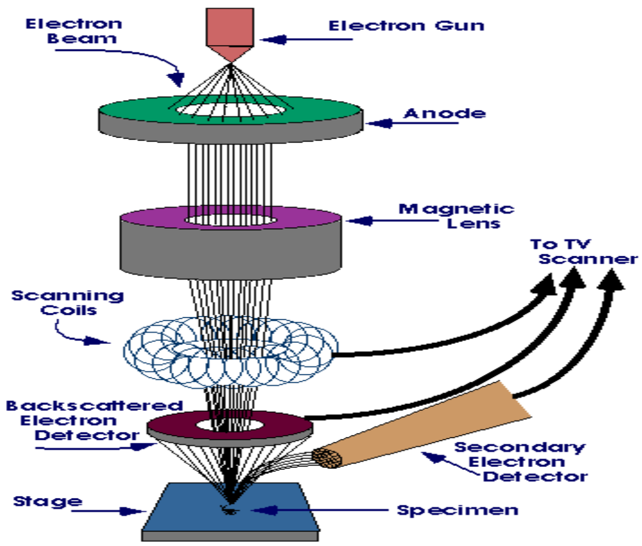
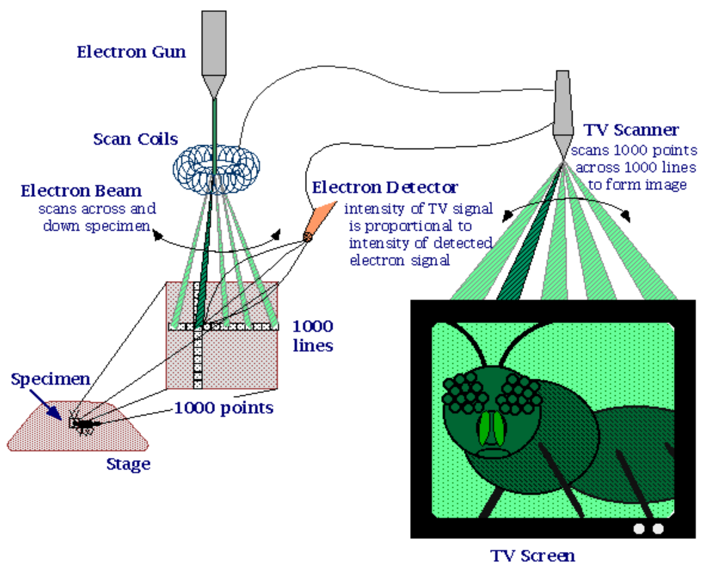


Fig. Electron microscope



Types of Electron microscopes

Transmission Electron Microscope (TEM)

TEM consists of a cylindrical tube about 2 meters long. The tube contains vacuum where the specimen is located. This is because the molecules of gases, such as those in air absorb electrons. TEM works by emitting electrons from a cathode, then accelerating them through an anode, after which the electrons pass through an aperture into the vacuum tube.

The image can be photographically recorded by exposing a photographic film or plate directly to the electron beam, or a high-resolution phosphor may be coupled by means of a fiber optic light-guide to sensor of a CCD (charged coupled device) camera. The image detected by the CCD may be displayed on a monitor or computer.

TEM applications

- TEMs provide topographical, morphological, compositional and crystalline information.
- It is useful in the study of crystals and metals, but also has industrial applications.
- TEMs can be used in semiconductor analysis and the manufacturing of computer and silicon chips.
- Tech giants use TEMs to identify flaws, fractures and damage to micro-sized objects; this data can help and fix problems and/or help to make a more durable efficient product.
- Colleges and universities can utilize TEMs for research and studies.

Scanning Electron Microscope (SEM)

The SEM is designed for direct study of the surfaces of solid objects. By scanning with an electron beam that has been generated and focused by the operation of the microscope, an image is formed in the same way as a TV.

Unlike the TEM, where electrons of the high voltage beam from the image of the specimen, the Scanning Electron Microscope (SEM) produces images by detecting low energy secondary electrons which are emitted from the surface of the specimen due to excitation by the primary electron beam.

Applications of SEM

- Image morphology of samples
- Image composition and finding some bonding differences (through contrast and using backscattered electrons)
- Image molecular probes: metals and fluorescent probes.
- Wet and dry samples while viewing them .
- View frozen material (in an SEM with a cryostage)

Electron microscopy application areas

1. **Semiconductor and data storage , Biology and life sciences**
2. **Research , Industry**

End of unit one Exercises

Show that the photons in a 1240nm infrared light beam have energies of 1.00*eV*.

1. A metal of work function 2.50eV is irradiated with light of an unknown frequency. The maximum velocity of the photo electrons is $1.14 \times 10^{6} ms^{-1}$ a) Calculate the maximum wavelength of the incident radiation. b) Explain what do you understood by Work function.
   c) Compute the energy of a photon of blue light of wavelength 450nm.
2. To break a chemical bond in the molecules of human skin and thus cause sunburn, photon energy of about $3.50 eV$ is required. To what wavelength does this correspond?
3. The photoelectric threshold wave length of silver is 2762 A^{O}, when the silver surface is illuminated with ultraviolet light of wave length 2000A^{o} calculate:

a) The maximum kinetic energy of the ejected electron in Electron-volts (eV)
b) The maximum velocity of the electrons
c) The stopping potential (V_o) in Volts for the electrons

4. The work function of sodium metal is $2.3eV$. What is the longest-wavelength light that can cause photoelectron emission from sodium?

5. What potential difference must be applied to stop the fastest photoelectrons emitted by a nickel surface under the action of ultraviolet light of wavelength 200nm? The work function of the nickel is $5.01eV$

UNIT II: SIMPLE HARMONIC MOTION

Introduction

A motion which repeats itself in equal fixed time intervals is **periodic motion**. Examples are ***planet in its orbit, water wave, pendulum, oscillating spring, and molecular vibrations***.

A simple harmonic motion (S.H.M) is defined as follows: Simple harmonic motion is a periodic motion whose the acceleration (or force) is directly proportional to its displacement from a fixed point and is always directed towards that point. Mathematically $a \propto -x$ (the negative sign is due to the fact that a and x are in opposite directions).

2. **Kinematics and simple harmonic motion**

Simple harmonic motion is a type of motion where the restoring force F is directly proportional to the displacement x and acts in the direction opposite to that of displacement.

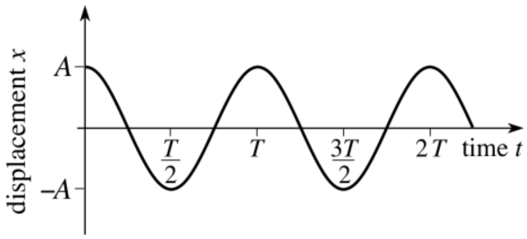
That is $F \propto x$; $F = -kx$ but the force F is given by $F = ma$ thus $ma = -kx$ **;** this is called the force law of simple harmonic motion.

$a = \frac{(-kx)}{m}$; $a = -\frac{k}{m}x$. The negative sign signifies that the force and acceleration are always pointing back towards the mean position respectively. The values k and m are constants.

Taking the SHM as a circular periodic motion $F = m\omega^2 x - kx = 0$; then $m\omega^2 x = kx$; so $\omega^2 = \frac{k}{m}$, this means that the acceleration is directly proportional to the displacement from a fixed point and it is always directed towards this point $a \propto -x$.

1.1. Definition of terms

Time period or periodic time T : it is the time taken for the particle to complete one oscillation, that is, the time taken for the particle to move from its starting position and return to its original position **Frequency** f means how many oscillations occur in one second. the frequency is expressed by; $f = \frac{1}{T}$ **Amplitude** A is the maximum displacement of the particle from its resting position or mean position.



From the graph, the displacement can be represented as $x = A\sin\omega t$.

Angular velocity ω : angular velocity is the rate of change of angular displacement. It is measured in $rads^{-1}$. This is related to periodic time according to equation $\omega = \frac{2\pi}{T}$.

Linear velocity v is the rate of change of linear displacement. It is measured in ms^{-1} ; $v = \frac{dx}{dt}$.

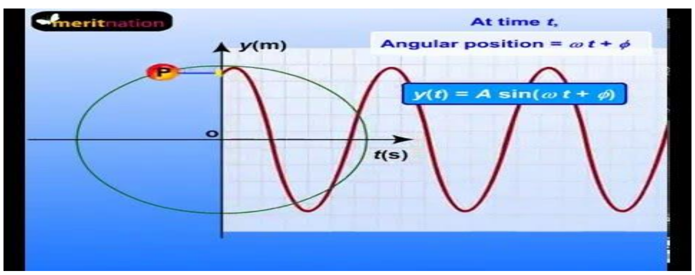
Linear acceleration a of a particle is the rate of change of linear velocity of that particle with time. It is measured in ms^{-2} ; $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$.

3. **Equation of SHM**

The equation of simple harmonic motion is derived based on the conditions necessary for periodic motion to be simple harmonic.

a) Displacement

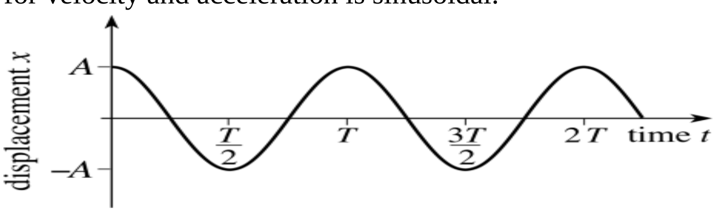
If the circular motion is converted into SHM, we have:



$y(t) = r\sin(\omega t + \phi)$ (Along the vertical axis i.e. vertical oscillations).

$x(t) = r\cos(\omega t + \phi)$ (Along the horizontal axis i.e. horizontal oscillations).

A graph of variation of the displacement of N with time on the horizontal axis i.e. its time trace like those for velocity and acceleration is sinusoidal.



b) Velocity The velocity of N is the component of P's velocity parallel to AB i.e. $-v\sin\theta = -\omega r\sin\theta$ or $v = \frac{dx}{dt} = \frac{d(r\cos\omega t)}{dt} = \frac{d(\cos\omega t)}{dt} = -r\omega\sin\omega t$.

The variation of the velocity of N with displacement x is given by $\pm\omega r\sqrt{1-\cos^2\theta}$ since $\sin\theta = \sqrt{1-\cos^2\theta}$ from $\cos^2\theta + \sin^2\theta = 1$

As $\cos\theta = \frac{x}{r}$, $v = \pm\omega r\sqrt{1-\left(\frac{x}{r}\right)^2}$: $v = \pm\omega r\sqrt{1-\left(\frac{r\cos\theta}{r}\right)^2} = \pm\omega r\sqrt{\frac{r^2 - r^2\cos^2\theta}{r^2}}$

$$v = \pm\omega\sqrt{\frac{r^4 - r^4\cos^2\theta}{r^2}} = \pm\omega\sqrt{\frac{r^2\left(r^2 - r^2\cos^2\theta\right)}{r^2}}$$

$$v = \pm\omega\sqrt{r^2 - r^2\cos^2\theta} = \pm\omega\sqrt{r^2 - (r\cos\theta)^2}$$

$$v = \pm\omega\sqrt{r^2 - x^2}$$

The velocity of N is maximum when $x = 0$ i.e. $v_{max} = \pm\omega r$, and it is zero when $x = \pm r$.

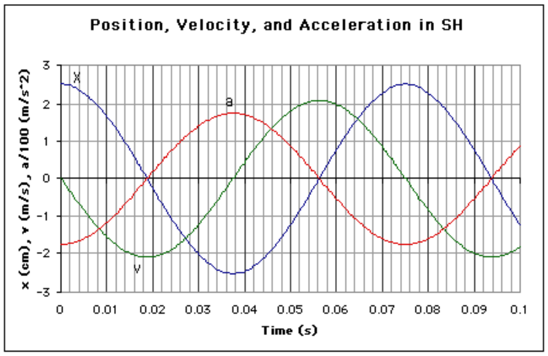
Note that when the velocity is zero, the acceleration is maximum and vice versa. We say that there is a ***phase difference of a quarter of a period*** $\left(\frac{T}{4}\right)$ between the velocity and the acceleration. The phase difference between the displacement and the acceleration is a half of a period $\left(\frac{T}{2}\right)$.

c) Acceleration

The acceleration is the rate of change of velocity $a = \frac{d(-r\omega\sin\omega t)}{dt} = -r\omega^2\cos\omega t$ i.e. $a = -x\omega^2$ where $x = r\cos\theta$.

The variation of acceleration a with the displacement x

x	0	$+r$	$-r$
a	0	$-\omega^2 r$	$+\omega^2 r$



d) Expression for angular velocity ω and the period

For a particular SHM, the angular speed ω is constant and so the period T is constant and independent of the amplitude r of the oscillation. If the amplitude increases, the body travels faster and so the period T remains unchanged.

Consider the equation for simple harmonic motion $a = -\omega^2 x$, we can ignore the sign and obtain $a = \omega^2 x$ and then $\omega^2 = \frac{a}{x}$. Multiplying both numerator and denominator by the mass m, we get $\omega^2 = \frac{ma}{mx} = \frac{\frac{ma}{x}}{m}$. The ratio $\frac{ma}{x}$ is the force per unit displacement i.e. the force causing the displacement x, so $\omega^2 = \frac{\frac{F}{x}}{m}$ and $\omega = \sqrt{\frac{\frac{F}{x}}{m}}$ or $\omega = \sqrt{\frac{force\ per\ unit\ displacement}{mass\ of\ oscillating\ system}}$. The force per unit displacement is also called the spring constant and it is denoted by k then $\omega = \sqrt{\frac{k}{m}}$ i.e. $\omega^2 = \frac{k}{m}$. The period of the SHM is the time required to complete one revolution (turn, cycle, 4 quadrants, angular displacement of 2π) $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{k}{m}}}$ that is $T = 2\pi\sqrt{\frac{m}{k}}$.

It is clear that the period T increases with the increase of mass of the oscillating system and/or the decrease of the force per unit displacement (spring constant).

An expression for the angular speed can also be deduced as follow for a system spring-mass:

The force $F = ma = m\omega^2 x$. This force is balanced by the restoring force $F = kx$. Equating the two equations we get: $m\omega^2 x = kx$. Making ω the subject of the equation, we get $\omega = \sqrt{\frac{k}{m}}$.

The frequency is the number of complete cycles per unit time $f = \frac{1}{T}$ i.e. $f = \frac{\omega}{2\pi}$.

By equating both expressions for the acceleration $a = \frac{d^2x}{dt^2}$ and $a = -\omega^2 x$ gives $\frac{d^2x}{dt^2} = -\omega^2 x$ i.e. $\frac{d^2x}{dt^2} + \omega^2 x = 0$; this is an differential equation of simple harmonic motion. Its solution is $x = r\sin(\omega t + \phi)$ where x is the displacement and r is the amplitude.

4. **Simple harmonic oscillators**

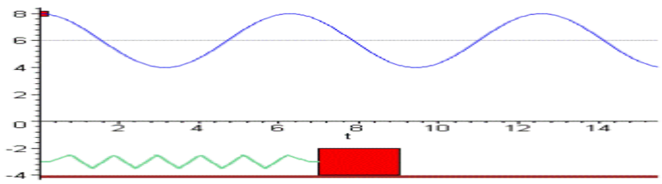
   A simple harmonic oscillator is a physical system in which a particle oscillates above and below a mean position at one or more characteristic frequencies.

   Mass on a spring (elastic pendulum)

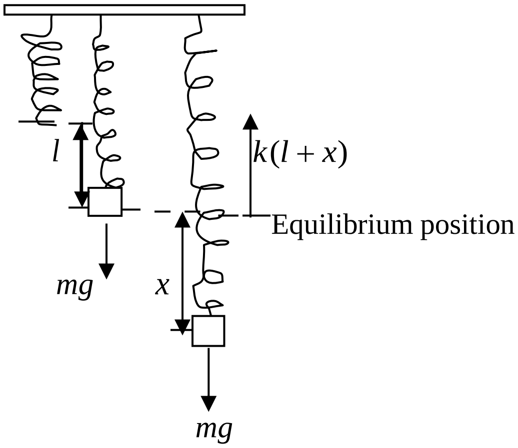
a) Period of oscillations

The extension of a spiral spring which obeys Hooke's law is directly proportional to the extending tension. A mass m attached to the end of a spring exerts a downwards tension mg on it and if it stretches it by an amount l, then if k is the spring constant, the stretching tension is kl therefore $mg = kl$.

Horizontal oscillations: $x(t) = r\cos(\omega t + \rho)$



Vertical oscillations : $y(t) = r\sin(\omega t + \rho)$



If the mass is pulled down a further distance x below its equilibrium position, the stretching tension acting downwards is $k(l + x)$ which is also the tension in the spring acting upwards. Hence the resultant restoring upwards force on the mass is $F = k(l + x) - mg$

$$F = kl + kx - mg$$
$$F = kl + kx - kl$$
$$F = kx.$$

When the mass is released, it oscillates up and down. If it has an acceleration a at extension x then by Newton's second law, $-kx = ma$

$$a = -\frac{kx}{m} = -\frac{k}{m}x = -\omega^2 x \text{ so } \omega^2 = \frac{k}{m}$$

The period $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{\sqrt{\dfrac{k}{m}}}$ i.e. $T = 2\pi\sqrt{\dfrac{m}{k}}$ or $T = 2\pi\sqrt{\dfrac{e}{g}}$ where ℓ is the extension.It follows that

$T^2 = \dfrac{4\pi^2 m}{k}$. If the mass m is varied and the corresponding periods T found, a graph of T^2 against m, is a straight line but it does not pass through the origin as we might expect from the above equation. This is because of the mass of the spring itself being neglected in the above derivation. Its effective mass and a value of g can be found experimentally.

Measurement of g and effective mass of spring. Let m_s be the effective mass of the spring; then $T = 2\pi\sqrt{\left(\frac{m+m_s}{k}\right)}$ but $mg = kl$ and $m = \frac{kl}{g}$.

Substituting for m in the first equation and squaring, we get $T^2 = 4\pi^2\left(\frac{m+m_s}{k}\right)$

$$T^2 = 4\pi^2\left(\frac{\frac{kl}{g}+m_s}{k}\right) = 4\pi^2\left(\frac{l}{g}+\frac{m_s}{k}\right)$$

$$T^2 = \frac{4\pi^2 l}{g} + \frac{4\pi^2 m_s}{k}$$

$$\frac{4\pi^2 l}{g} = T^2 - \frac{4\pi^2 m_s}{k} \text{ and } \frac{4\pi^2 m_s}{k} = T^2 - \frac{4\pi^2 l}{g}$$

$$g = \frac{4\pi^2 l}{T^2 - \frac{4\pi^2 m_s}{k}}$$

$4\pi^2 l = g\left(T^2 - \frac{4\pi^2 m_s}{k}\right)$ and then $l = \frac{g}{4\pi^2}\left(T^2 - \frac{4\pi^2 m_s}{k}\right)$

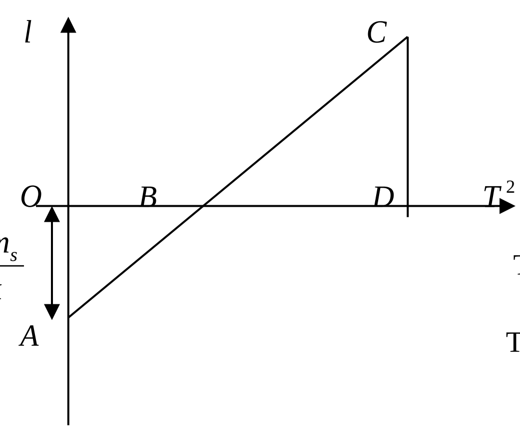
$4\pi^2 m_s = k\left(T^2 - \frac{4\pi^2 l}{g}\right)$ and $m_s = \frac{k}{4\pi^2}\left(T^2 - \frac{4\pi^2 l}{g}\right)$

By measuring i) the static extension l and

ii) The corresponding period T using several different masses in turn, a graph of l against T^2 can be drawn. It is a straight line of slope $\frac{g}{4\pi^2}$ and intercept $\frac{gm_s}{k}$ on the axis l.

This enables g and m_s to be found. Theory suggests that the effective mass of a spring is about one third of its actual mass.

Graph



Intercept $OA = \frac{gm_s}{k}$

The slope $S = \frac{CD}{BD}$ i.e. $S = \frac{g}{4\pi^2}$

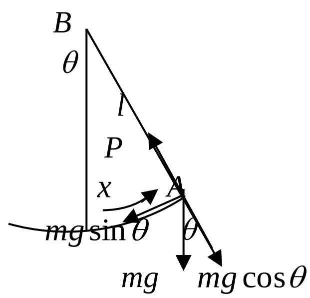
The gravity $g = 4\pi^2 S$.

The effective mass of the spring is $m_s = \frac{k\overline{OA}}{g}$ that is $m_s = \frac{k\overline{OA}}{4\pi^2 S}$ where $\overline{OA}$ is the magnitude of l -intercept.

3.1. Simple pendulum

a) Period of oscillation

A simple pendulum consists of a small bob (in theory a particle) of mass m suspended by a light inextensible thread of length l from a fixed point. If the bob is drawn aside slightly and released, it oscillates to and fro in a vertical plane along the arc of a circle. It describes a simple harmonic motion about its equilibrium position.



Resolving mg radially and tangentially at A, we see that the tangential component $mg\sin\theta$ is the unbalanced restoring force acting towards O. If a is the acceleration of the bob along the arc at A due to $mg\sin\theta$ then the equation of motion of the bob is $-mg\sin\theta = ma$.

When θ is small, $\sin\theta = \theta$ in radians and $x = l\theta$; $\theta = \frac{x}{l}$.

$$-mg\theta = -mg\frac{x}{l} = ma$$

$$-g\frac{x}{l} = a$$

$$-\frac{g}{l}x = -\omega^2 x \text{ (where } \omega^2 = \frac{g}{l}\text{)}$$

The motion of the bob is SHM if the oscillations are of small amplitudes i.e. $\theta \le 10^0$. The period T is given by $T = \frac{2\pi}{\omega}$, therefore $T = 2\pi\sqrt{\frac{l}{g}}$.

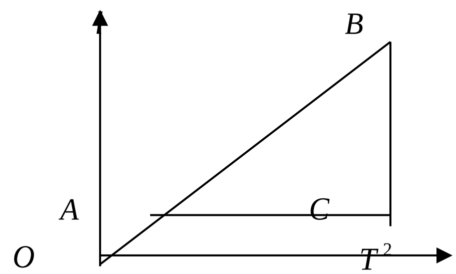
The period T is independent of the amplitude and the mass, and at a given place on the Earth's surface where g is constant, it depends only on the length l of the pendulum.

b) Measurement of g by a method of simple pendulum

A fairly accurate determination of g can be made by measuring the periods T for different values of l plotting a graph of l against T^2.

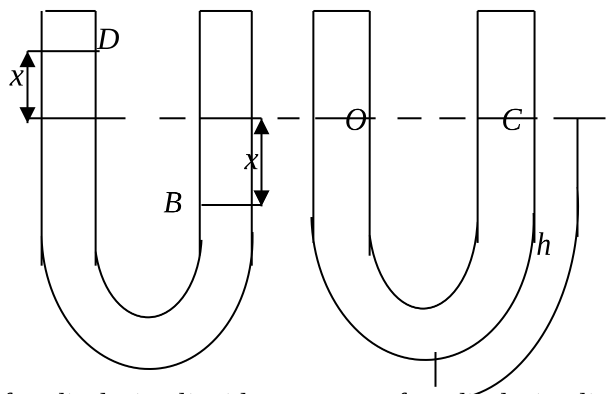
A straight line AB is then drawn so that the points are evenly distributed about it. It should pass through the origin and its slope $S = \frac{BC}{CA}$ gives an average value of $\frac{l}{T^2}$ from which g can be calculated. Since $T = 2\pi\sqrt{\frac{l}{g}}$, $T^2 = 4\pi^2 \times \frac{l}{g}$

$g = 4\pi^2 \times \frac{l}{T^2} \Leftrightarrow g = 4\pi^2 \times \frac{BC}{CA}$ then $g = 4\pi^2 S$.



3.3. Liquid in a U-tube

Consider a U-tube filled with a liquid. If the liquid on one side of a U-tube is depressed by blowing gently down that side, the level of the liquid will oscillate for a short time about the respective positions O and C before finally coming to rest.



After displacing liquid Before displacing liquid

As shown in figure, B is x units below the original level C , and D is x units above the original level O . Here x is displacement of the fluid caused by blowing into one arm of the U-tube.
Usually, pressure in liquid is given by $P = density \times acceleration\ due\ to\ gravity \times height$

$P = \rho gh$

Excess pressure exerted in the liquid will store some energy to restore the position of the liquid and is given by: $P = density \times acceleration\ due\ to\ gravity \times excess\ height$

$P = \rho g \times 2x = 2\rho gx$

Also the pressure $P = \frac{F}{A}$ and $F = PA$

The force on the liquid $F_1 = 2\rho gxA$

$\omega = \frac{2\pi}{T} = \sqrt{\frac{g}{h}}$

From Newton's second law of motion, the resultant force on the liquid is given by $F_2 = ma$ where m is the mass of the oscillating liquid.

As $Mass = Volume \times density$; and $Volume = Area \times Lenght$; we get $m = \rho V$ and $V = Al$ with $l = 2h$.

The mass $m = 2\rho Ah$ and $F_2 = 2\rho Aha$

F_1 and F_2 are equal and opposite to each other; $F_2 = -F_1$

$2\rho Aha = -2\rho gxA$

$a = -\frac{gx}{h}$ where g and h are constant.

Comparing expressions $a = -\omega^2 x$ and $a = -\frac{gx}{h}$, we get $\omega^2 = -\frac{g}{h}$

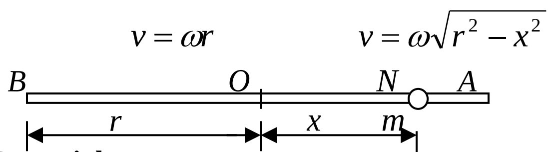
$T = 2\pi\sqrt{\frac{h}{g}}$; This is an expression for the period of a SHM of the liquid in a U-tube

4. Energy of SHM

There is a constant interchange of energy between the kinetic and potential forms, and if the system does not work against resistive forces (is undamped) its total energy is constant.

a) **Kinetic energy**

The velocity of a particle N of mass m at a distance x from its centre of oscillation is $v = \pm\omega\sqrt{r^2 - x^2}$. Kinetic energy at displacement x is $K.E = \frac{1}{2}m\omega^2\left(r^2 - x^2\right)$



b) **Potential energy**

As N moves out from O towards A (or B) work is done against the force trying to restore it to O. So N loses kinetic energy and gains potential energy. When $x = 0$, the restoring force is zero; at displacement x, the force is $m\omega^2 x$ (since the acceleration has magnitude $\omega^2 x$). Therefore average force on N while moving to displacement x, is $F = \frac{1}{2}m\omega^2 x$.

The work done= $average\ force \times displacement\ in\ direction\ of\ force.$

$W = \frac{1}{2}m\omega^2 x \times x = \frac{1}{2}m\omega^2 x^2$. The potential energy at displacement x is $P.E = \frac{1}{2}m\omega^2 x^2$

c) **Total energy of a SHM**

E4

At displacement x we have, total energy $= K.E + P.E = \frac{1}{2}m\omega^2\left(r^2 - x^2\right) + \frac{1}{2}m\omega^2 x^2$

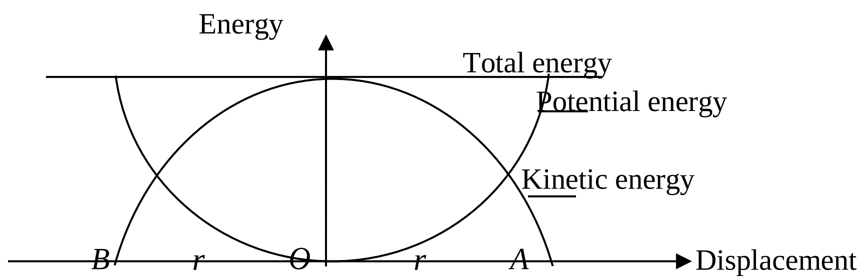
Total energy $= \frac{1}{2}m\omega^2\left(r^2 - x^2 + x^2\right) = \frac{1}{2}m\omega^2 r^2$ thus $E_t = \frac{1}{2}m\omega^2 r^2$.

This is constant, it does not depend on the displacement x and it is directly proportional to the product of i) the mass

ii) The square of the frequency

iii) The square of the amplitude.

Variation of kinetic energy, potential energy and total energy with displacement



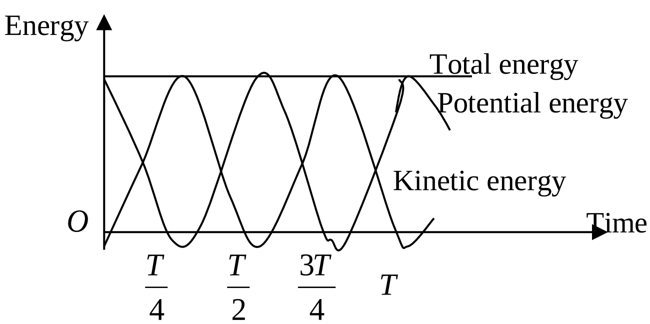
In simple pendulum, all the energy is kinetic as the bob passes through the center of oscillation and at the top of the swing it is all potential.

d) Variation of kinetic energy and potential energy with time

These energies vary with time as shown by the graphs.

The kinetic energy $= \frac{1}{2}mv^2 \Rightarrow K.E = \frac{1}{2}m\omega^2 r^2 \sin^2 \omega t$; (since $v = -\omega r \sin \omega t$)

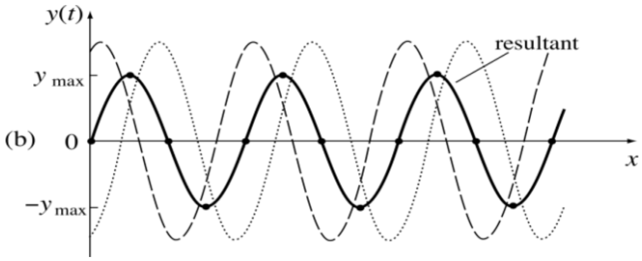
The potential energy $\frac{1}{2}m\omega^2 x^2 \Rightarrow P.E = \frac{1}{2}m\omega^2 r^2 \cos^2 \omega t$; ($x = r\cos\omega t$)



5. Superposition of harmonic motions with the same frequency

Superposition of simple harmonic motions consists of the propagation of two (or more) SHM travelling through the same medium at the same time. **The principle of superposition states** that ***"when two or more simple harmonic motions travelling in a medium superpose upon each other, then the resultant displacement at any instant is equal to the vector sum of the displacement due to individual simple harmonic motion".***

The resultant of the superposition of simple harmonic motions is the linear combination of individual simple harmonic motions.



Consider two special cases:

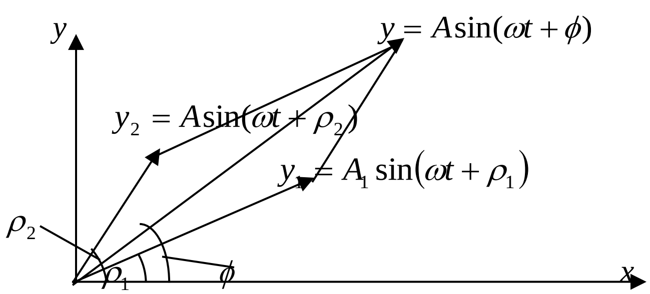
1. If $\Delta\rho = 0$ we say that the two motions are ***in phase***. Then the resultant motion is $x = (A_1 + A_2)\cos\omega t$. The equation shows that the resultant motion is also SHM with the same angular frequency. The motion has amplitude equal to the sum of the amplitudes of the two motions; that is $A = A_1 + A_2$.
2. When $\Delta\rho = p$, we have $x_2 = A_2\cos(\omega t + p)$ i.e. $x_2 = -A_2\cos\omega t$. The resultant motion is $x = A_1\cos\omega t - A_2\cos\omega t$ i.e. $x = (A_1 - A_2)\cos\omega t$ which shows that the resultant motion is SHM with the same angular frequency and amplitude equals to the difference of the amplitudes of the two motions; that is for that reason we say that the motions are in ***opposition.***

   In general case, where the phase difference is arbitrary; the resultant motion is also SHM with the same angular frequency ω and amplitude given $A = \sqrt{\left(A_1^2 + A_2^2 + 2A_1A_2\cos\delta\right)}$. It can be seen and form a **fixed angle** $\Delta\rho$.

   Superposition methods

- **Fresnel vectors method**

This method is used for the superposition of any two motions of the same frequency.



This method uses vectors addition where the resultant amplitude is given by cosine rule and the phase difference from the arctangent.

$x_1 = A_1\cos(\omega t + \rho_1)$ i.e. $y_1 = A_1\sin\left(\frac{\pi}{2} - (\omega t + \rho_1)\right)$ and

$x_2 = A_2\cos(\omega t + \rho_2)$ i.e. $y_2 = A_2\sin\left(\frac{\pi}{2} - (\omega t + \rho_2)\right)$

The resultant amplitude is $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\rho_2 - \rho_1)}$

The phase difference of the resultant motion is $\phi = \arctan\left(\frac{A_1 \sin\rho_1 + A_2 \sin\rho_2}{A_1 \cos\rho_1 + A_2 \cos\rho_2}\right)$

The resultant motion is $y = A\sin(\omega t + \phi)$.

- **Simpson method**

It is applicable for two motions of the ***same amplitude*** and ***same frequencies*** but with a phase difference.

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) \qquad \cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right) \qquad \cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

Theoretical examples:

$y_1 = A\sin(\omega t + \rho_1)$ and $y_2 = A\sin(\omega t + \rho_2)$

$$y = y_1 + y_2 = A\sin(\omega t + \rho_1) + A\sin(\omega t + \rho_2)$$

$$y = 2A\sin\left(\frac{\omega t + \rho_1 + \omega t + \rho_2}{2}\right)\cos\left(\frac{\omega t + \rho_1 - \omega t - \rho_2}{2}\right)$$

$y = 2A\sin\left(\omega t + \frac{\rho_1 + \rho_2}{2}\right)\cos\left(\frac{\rho_1 - \rho_2}{2}\right)$. The resultant amplitude is $A_R = 2A\cos\left(\frac{\rho_1 - \rho_2}{2}\right)$

The phase difference of the resultant motion is $\phi = \frac{(\rho_1 + \rho_2)}{2}$.

For the motions expressed in cosine function, $x_1 = A\cos(\omega t + \rho_1)$ and $x_2 = A\cos(\omega t + \rho_2)$

$x = 2A\cos\left(\omega t + \frac{\rho_1 + \rho_2}{2}\right)\cos\left(\frac{\rho_1 - \rho_2}{2}\right)$. The resultant amplitude is $A_R = 2A\cos\left(\frac{\rho_1 - \rho_2}{2}\right)$

The phase difference of the resultant motion is $\phi = \frac{(\rho_1 + \rho_2)}{2}$

Applications of superposition principle

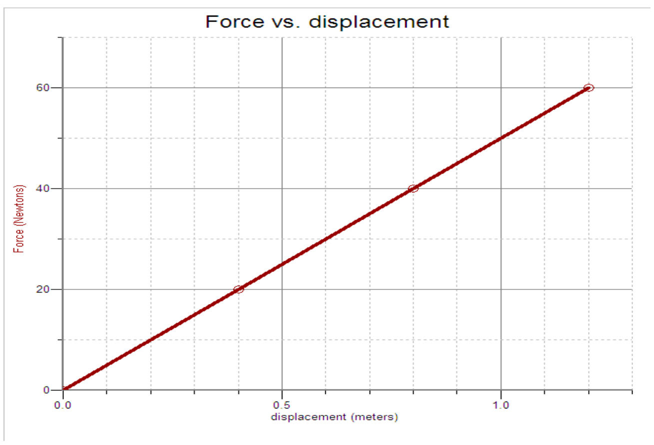
Superposition principle can be applied to simple harmonic motions and waves in the following phenomenon:

1. ***Diffraction*** of secondary wavelets originating from coherent sources on the same wavefront
2. ***Stationary waves*** result from the superposition of two waves of the same amplitude and frequency travelling at the same speed in opposite directions.
3. ***Interference*** results from the superposition of coherent waves from identical sources traveling in the same medium.
4. ***Beats***: two wavestrains of close frequency travelling in the same direction at the same speed.

End unit two exercises

1. Define the following terms:
a) amplitude b) equilibrium position c) frequency d) Hooke's law e) ideal spring
f) mechanical resonance g) period h) periodic motion i) restoring force
j) simple harmonic motion

2. A mass of 0.5 kg oscillates on the end of a spring on a horizontal surface with negligible friction according to the equation $x = A\cos(\omega t)$. The graph of F *vs.* x for this motion is shown below.



The last data point corresponds to the maximum displacement of the mass.

Determine the:

(a) angular frequency ω of the oscillation, (b) frequency f of oscillation,

(c) amplitude of oscillation, (d) displacement from equilibrium position ($x = 0$) at a time of 2 s.

3. A pendulum of mass 0.4 kg and length 0.6 m is pulled back and released from and angle of 10° to the vertical.
(a) What is the potential energy of the mass at the instant it is released. Choose potential energy to be zero at the bottom of the swing.

(b) What is the speed of the mass as it passes its lowest point?

This same pendulum is taken to another planet where its period is 1.0 second.

(c) What is the acceleration due to gravity on this planet

UNIT III: FORCED OSCILLATIONS AND RESONANCE

1. **Damped oscillations**

We know that in reality, a spring won't oscillate forever. Frictional forces will diminish the amplitude of oscillation until eventually the system is at rest. If we consider a pendulum oscillating in air, its amplitude decreases gradually to zero due to the resistive force arising from air; the motion is said to be **damped** by air resistance. Its energy becomes internal energy of the surrounding air.
Damping is the gradual decrease of amplitude of an oscillating system due to friction force (air resistance) and losses of energy. As work is being done against the **dissipating force**, energy is lost. Since energy is proportional to the amplitude, the amplitude decreases exponentially with time.

a. General characteristics of damped oscillations

1. Non conservative forces may be present: Friction is common non conservative force
   No longer an ideal system
2. The mechanical energy of the system diminishes in time, motion is said to be damped
3. The motion of a system can be decaying oscillations if damping is weak.
4. If damping is strong, motion may die away without oscillating
5. Still no driving force, once system has been started.
6. The amplitude decays at exponential rate so that $x(t) = x_0 e^{-\alpha t}$.

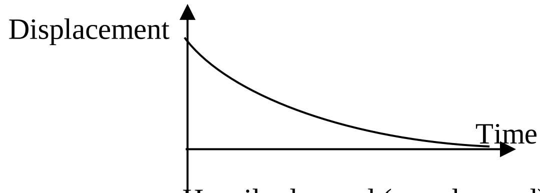
b. Types of damped oscillations

- **Underdamped (lightly damped) oscillations**
- **Overdamped (heavily damped) oscillations**
- **Critically damped oscillations.**

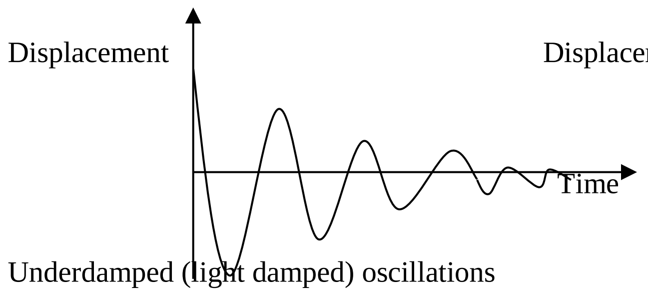
For **underdamped oscillations (slightly damped oscillations),** the system oscillates and the amplitude decays exponentially. The examples are acoustics:

- ***A percussion musical instrument*** (e.g. a drum) gives out a note whose intensity decreases with time. (Slightly damped oscillations due to air resistance).
- ***The paper cone of a loudspeaker*** vibrates, but is heavily damped so as to lose energy (sound energy) to the surrounding air.

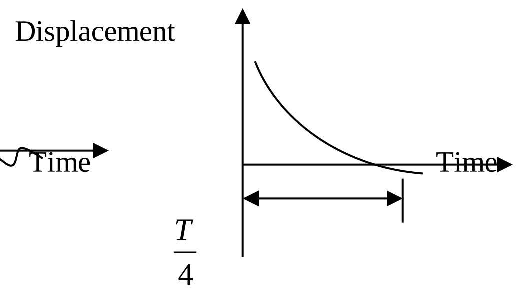
When a system is **heavily damped** no oscillations occur and the system returns very slowly to its equilibrium position. When the time taken for the displacement to become zero is a minimum $\frac{T}{4}$
The system is said to be **critically damped,** (with T the period of free oscillations).
Examples of critical damping are:
Shock absorber: it critically damps the suspension of the vehicle and so resists the setting up of vibrations which could make control difficult or cause damage. The viscous force exerted by the liquid contributes to this resistive force.
Electrical meters: they are critically damped (i.e. dead-beat) oscillators so that the pointer moves quickly to the correct position without oscillation.



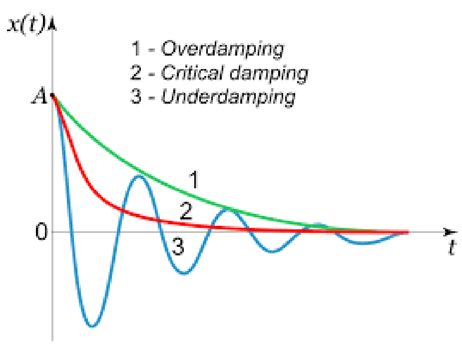
Heavily damped (overdamped) oscillations



Underdamped (light damped) oscillations



Critically damped oscillations.



c. Equation for damped oscillations

In terms of the derivatives, the equation is $m\frac{d^2x}{dt^2}+c\frac{dx}{dt}+kx=0$

where m is the mass, c is the constant which depends on the kind of liquid (fluid) and the shape of the mass.

The simplified equation is

$ma+cv+k=0$; where a is the acceleration, v is the velocity and k is the spring force.

The frictional force (damping force, viscous drag is proportional to the velocity) is $F_{damping}=cv$.

The restoring force is $F_{restoring}=kx$.

The net force $F_{net}=ma$ and Newton's second law of motion is written as follow:

$-kx-cv=ma$.

The oscillation has exponential decay terms which depend upon a damping coefficient. As $F_{damping} = cv$, then the damping coefficient c is given by $\lambda = \frac{c}{2m}$ .The parameter λ depends on the damping coefficient c.

By differentiating $m\frac{dx^2}{dt^2} + c\frac{dx}{dt} + kx = 0$, given that $x = x_0 e^{-\alpha t}$, the equation becomes: $m\frac{d^2(x_0 e^{-\alpha t})}{dt^2} + c\frac{d(x_0 e^{-\alpha t})}{dt} + k(x_0 e^{-\alpha t}) = 0$. As $\frac{dx}{dt} = -\alpha x_0 e^{-\alpha t}$ and $\frac{d^2 x}{dt^2} = \alpha^2 x_0 e^{-\alpha t}$; then we get $m\alpha^2 x_0 e^{-\alpha t} - c\alpha x_0 e^{-\alpha t} + kx_0 e^{-\alpha t} = 0$.

By simplifying the equation is $m\alpha^2 - c\alpha + k = 0$; and by solving this quadratic equation we get $\alpha = \frac{c \pm \sqrt{c^2 - 4mk}}{2m}$.

The general solutions for the equations of damped oscillations

- Concerning the ***decay of amplitude***at exponential rate,

If $x = \frac{1}{2}x_0$, then $t = T_{\frac{1}{2}}$. $\frac{1}{2}x_0 = x_0 e^{-\lambda t}$ i.e. $\frac{1}{2} = e^{-\lambda t}$, $\ln\left(\frac{1}{2}\right) = -\lambda t$, $t = \frac{\ln\left(\frac{1}{2}\right)}{-\lambda} \Rightarrow T_{\frac{1}{2}} = \frac{\ln(2)}{\lambda}$ this is the ***half-life***: time required for the initial amplitude of oscillations to decay to its half (to be reduced to its half) i.e. $x = \frac{1}{2}x_0$.

- If $c^2 \prec 4mk$ ($c^2 - 4mk \prec 0$, Δ is negative): ***(underdamped) light damped oscillations.***

The restoring force is large compared to the damping force; the system oscillates with decaying amplitude.

The general solution of the equation is $X_{underdamed} = Ae^{-\lambda t}\cos(\omega' t + \phi)$ i.e. $X(t) = Ae^{-\frac{ct}{2m}}\cos(\omega' t + \phi)$

The parameter λ depends on the damping coefficient c, and it is given by $\lambda = \frac{c}{2m}$.

The damping coefficient c is small relative to m and k i.e $c \prec 2\sqrt{mk}$.

The parameter ω' is the new oscillation frequency such that $\omega' = \sqrt{\frac{k}{m} - \frac{c^2}{4m^2}} = \sqrt{\omega^2 - \lambda^2}$.

The new angular frequency ω' is slightly less than the initial angular frequency ω.

The decay time $\tau = \frac{m}{c}$.

The amplitude decays in time as $e^{-\frac{t}{2\tau}}$, and energy is proportional to A^2 decays as $e^{\frac{t}{\tau}}$.

- If $c^2 \succ 4mk$ ($c^2 - 4mk \succ 0$, Δ is positive): ***overdamped (heavy damped) oscillations***

The damping coefficient c is large relative to m and k i.e. $c \succ 2\sqrt{mk}$
The damping force is much stronger than the restoring force
The amplitude dies away as a modified exponential.

The equation is: $X_{overdamped}(t) = Ae^{-\lambda t}$ (λ is always less than ω i.e. $\lambda \prec \omega$) or $X(t) = Ae^{-\frac{ct}{2m}}$ where $\lambda = \frac{c}{2m}$.

- If $c^2 = 4mk$ ($c^2 - 4mk = 0$, $\Delta = 0$): ***critical damped oscillations***.

It is the transition from over-damping to under-damping and vice-versa.
There is a large value of λ .
The damping coefficientis $c = 2\sqrt{mk}$.
The restoring force and damping force are comparable in effect.
The system cannot oscillate; the amplitude dies away exponentially.

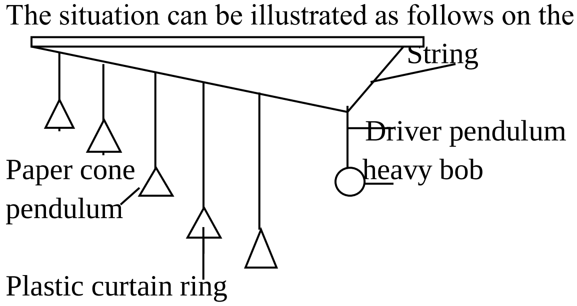
$X_{critical\ \ damping}(t) = (A_1 + A_2)e^{-\lambda t}$ or $X_{critical\ \ damping}(t) = (A_1 + A_2 t)e^{\frac{-ct}{2m}}$.

- About the decay of energy of the damped system,

For small damping constant c the mechanical energy E of the oscillator is given by $E(t) \approx \frac{1}{2}kx_0^2 e^{\frac{-ct}{m}}$ i.e. $E(t) \approx \frac{1}{2}kx_0^2 e^{-2\lambda t}$.

2. **Natural frequency of vibration and forced oscillation**
   A forced oscillator is one for which a force continually or repeatedly applied to keep the oscillation going. E.g. a swing pushed each time it reaches a certain point, behaves as a forced oscillator and will continue to swing for as long as energy is supplied. ***Forced vibration is a vibration in which a system is involuntary compelled to vibrate.***

The situation can be illustrated as follows on the arrangement known as Barton's pendulum.



The pendulum whose length equals that of the driver has the greatest amplitude, its natural frequency of oscillation is the same as the frequency of the driving pendulum. This is an example of resonance and the driving oscillator then transfers its energy most easily to the other system i.e. the paper cone pendulum of the same length. The natural frequency that the swing wants to oscillate at is resonant frequency.

We define ***the natural frequency*** *as the frequency at which a system vibrates when set in free vibration* while ***the forcing frequency is*** *the frequency of an external periodic force applied to a system* and ***forced frequency*** *is the frequency of vibration of the system which has been forced to vibrate.*

a. Characteristics of forced oscillations

*An external driving force starts oscillations in a stationary system
*The amplitude remains constant (or grows) if the energy input per cycle exactly equals (or exceeds) the energy loss from damping

Eventually, $E_{driving} = E_{lost}$ and a steady-state condition is reached

*Oscillations then continue with constant amplitude

*Oscillations are at the driving frequency ω_D .

*The oscillating driving force applied to a damped oscillator is $F_D(t) = F_0 \cos(\omega_D t + \phi')$

The net force is $F_{net} = F_D(t) - c\dfrac{dx(t)}{dt} - kx = m\dfrac{d^2x(t)}{dt^2}$

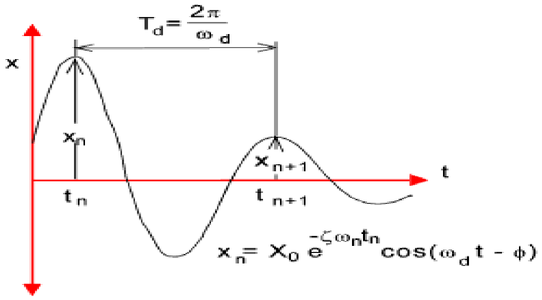
The solution of the equation of motion is $X(t) = A\cos(\omega_D t + \phi)$

where $A = \dfrac{\dfrac{F_0}{m}}{\sqrt{\left(\omega_D^2 - \omega_0^2\right)^2 + \left(\dfrac{c\omega_D}{m}\right)^2}}$ or $A = \dfrac{1}{m} \times \dfrac{F_0}{\sqrt{\left(\omega_D^2 - \omega_0^2\right)^2 + \left(\dfrac{c\omega_D}{m}\right)^2}}$ or again

$A = \dfrac{F_0}{\sqrt{m^2\left(\omega_D^2 - \omega_0^2\right)^2 + \left(\dfrac{c\omega_D}{m}\right)^2}}$ where ω_0 is the natural angular frequency and

ω_D is the driving angular frequency of external force.

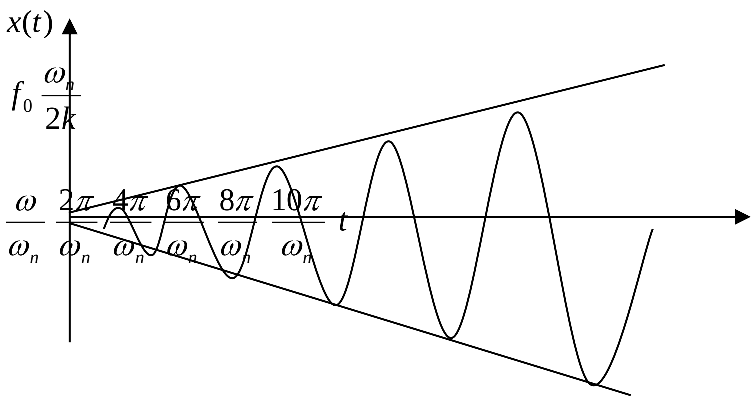
The amplitude A depends on how close ω_D is to natural frequency ω_0 .



b. Variation of amplitude on graph at forcing frequency close to natural frequency of vibration

The equation is $\dfrac{m}{k}\dfrac{d^2x}{dt^2} + \dfrac{\lambda}{k}\dfrac{dx}{dt} + x = \dfrac{1}{k}F_0 \sin\omega t$ or $m\dfrac{d^2x}{dt^2} + \lambda\dfrac{dx}{dt} + kx = F_0 \sin\omega t$.

The amplitude of vibration is strongly dependent on the frequency of excitation, and on the properties of the spring-mass system. If the forcing frequency is close to the natural frequency of the system, and the system is lightly damped, huge vibration amplitudes may occur. On the graph the situation is illustrated as follow:



c. Resonance

By definition, ***resonance is the increase in amplitude of oscillation of an electric or mechanical system exposed to a periodic force where frequency is equal to the natural frequency of the mechanical or electrical system.***

In physics, **resonance describes** a vibrating system or external force which drives (forces) another system to oscillate with greater amplitude at a specific preferential frequency. Increase of amplitude as damping decreases and frequency approaches resonant frequency of a driven damped simple harmonic oscillator. Frequencies at which the response amplitude is a relative maximum are known as the **system's resonant** frequencies or resonance frequencies. **At resonant frequencies**, small periodic driving forces have the ability to produce large amplitude oscillations. This is because the system stores vibrational energy.
Resonance occurs when a system is able to store and easily transfer energy between two or more different storage modes (such as kinetic energy and potential energy in the case of a pendulum). However, there are some losses from cycle to cycle, called damping. When damping is small, the resonant frequency is approximately equal to the natural frequency of the system, which is a frequency of unforced vibrations.

Applications and examples (types) of resonance in everyday life
Resonance occurs with all types of vibrations or waves. There are:

A washing machine
A washing machine may vibrate quite violently at particular speeds.
Breaking the glass using voice
You have heard the story of an opera singer who could shatter a glass by singing a note at its natural frequency. The singer sends out a signal of varying frequencies and amplitudes that makes the glass vibrate. At a certain frequency, the amplitude of these vibrations becomes maximum and the glass fails to support it and breaks it.

Breaking the bridge

The wind, blowing in gusts, once caused a suspension bridge to sway with increasing amplitude until it reached a point where the structure was over-stressed and the bridge collapsed.

Tuning circuit

The other example of useful resonance is the tuning circuit on a radio set. Radio waves of all frequencies strike the aerial and only the one which is required must be picked out.

Microwave Ovens

Microwave ovens use resonance. The frequency of microwaves almost equals the natural frequency of vibration of a water molecule. This makes the water molecules in food to resonate. This means they take in energy from the microwaves and so they get hotter. This heat conducts and cooks the food.

Magnetic resonance imaging (MRI)

The picture showing the insides of the body was produced using magnetic resonance imaging.

Mechanical resonance: is the tendency of a mechanical system to absorb more energy when the frequency of its oscillations matches the system's natural frequency of vibration than it does at other frequencies. It may cause violent swaying motions and even catastrophic failure in improperly constructed structures including bridges, buildings, trains and aircrafts.

Acoustic resonance: is related to sound waves when producing beats, musical intervals, and concert hall acoustics, distinctions between noise and music, and sound production by musical instruments.

Electrical resonance occurs in an electric circuit at a particular resonant frequency when the impedance of the circuit is at a minimum in a series circuit or at maximum in a parallel circuit (or when the transfer function is at a maximum) example: tuned circuits in radios and T.V. that allow radio frequencies to be selectively received.

Optical resonance: an optical cavity or optical resonator is an arrangement of mirrors that forms a standing wave cavity resonator for light waves.

Example: the creation of coherent light in a laser cavity.

Orbital resonance: it occurs when two orbiting bodies exert a regular, periodic gravitational influence on each other, usually due to their orbital periods being related by a ratio of two small integers.

Electromagnetic resonance: is a phenomenon produced by simultaneously applying steady magnetic field and electromagnetic radiation (usually radio waves) to a sample of electrons and then adjusting both the strength of magnetic field and the frequency of the radiation to produce absorption of the radiation.

Nuclear, atomic, particle, molecular resonance: nuclear magnetic resonance (NMR) is the name given to a physical resonance phenomenon involving the observation of specific quantum mechanical magnetic properties of an atomic nucleus in the presence of an applied external magnetic field

Resonance of quantum wave functions: the wave function describes the position and state of the electron and its square gives the probability density of electrons.

Tidal resonance: it occurs when the tide excites one of the resonant modes of the ocean. The effect is most striking when a continental shelf is about a quarter wavelength wide. Then an incident tidal wave can be reinforced by reflections between the coast and the shelf edge, the result producing a much higher tidal range at the coast.

d. Bandwidth and quality factor of a resonance curve

A resonance curve: is a graph which represents the variation of energy or displacement with frequency of forced oscillations.

A quality factor Q is a dimensionless parameter that describes how under-damped an oscillator or resonator is, or equivalently, characterizes a resonator's bandwidth relative to its center frequency. A high quality factor indicates a lower rate of energy loss relative to the stored energy of the oscillator, i.e. the oscillations die out more slowly. A pendulum suspended from a high quality bearing, oscillating in air, has a high quality factor while a pendulum immersed in oil has a low quality factor. $Q = \frac{Pstored}{Pdissipated}$ or $Q = \frac{f_0}{\Delta f}$. ***The bandwidth Δf is the width of the range of frequencies for which the energy is at least half its peak value or the displacement is a least $\frac{1}{\sqrt{2}}$ (0.7071 or 70.71%) of its amplitude***. The energy of oscillation, or the power dissipation, decays twice as fast, that is, as the square of the amplitude.

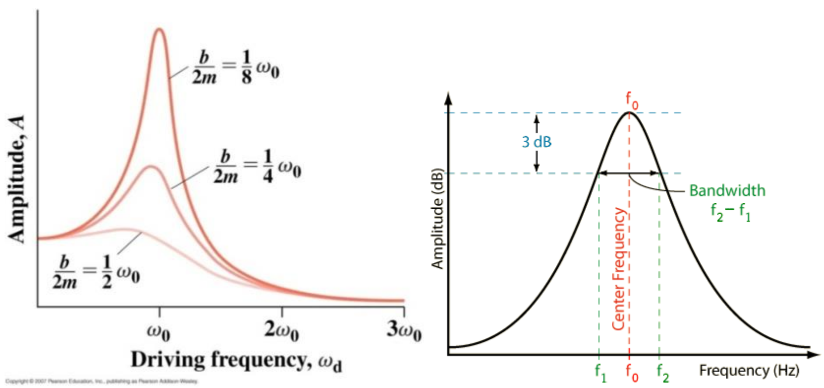


Figure: Resonance curve

Then, $\Delta f = f_2 - f_1$ and $Q = \frac{f_0}{f_2 - f_1}$ or $Q = \frac{f_0}{\Delta f}$.

e. Advantages of resonance

1. Hearing occurs when the ear drum (tympanic membrane) resonates to sound waves hitting it. Microphones and diaphragm in the telephone work in much the same way. Speakers in hifis, T.Vs etc use resonance as well.
2. Resonance is used in guitars and pianos as well as wood wind instruments to produce sound.
3. Resonance is also used to shatter gallstones in patients using ultrasound.
4. Magnetic resonance imaging (MRI) scans are used every day in hospitals for diagnostic purposes and also for industrial purposes.

f. Disadvantages of resonance

1. Negative effect of resonance could be the effect of waves hitting a rock face. The vibration of kinetic energy from the wave resonates through the rock face causing cracks and eventually great slabs of the cliff fall into the sea.
2. In a car crash, a passenger may be injured because his chest is thrown hard against the seat belt; the vibration can burst blood vessels.
3. If one is shot with a bullet, the resonance of the bullet hitting the body can cause liquefaction of the internal organs; this can also occur if you are near a loud explosion. Vibration of the explosion may apparently burst blood vessels and liquefy some of the organs.
4. Resonance can set a bridge swinging and destroy it. for that reason, anybody example the army men always break step and do not match over a bridge so that they do not risk hitting its resonance frequency.In 1940, Tacoma Narrows Suspension Bridge (Galloping Gertie) collapsed just a few months after its opening.
5. Shattering glass when a high pitched sound is played (like a singer's voice).
6. earthquakes and damage to buildings
7. Negative effect of resonance is when bridge builders get it wrong and the wind causes it to resonate at its own frequency causing it to tear its self apart.

Effect of resonance on a system

- Buildings driven by earthquakes
- Bridges under wind load
- All kind of radio devices, microwaves
- Vibrations at resonance can cause bursting of blood vessel
- In a car crash a passenger may be injured because their chest is thrown against the seat belt.
- The vibration of kinetic energy from the wave resonates through the rock face and causes cracks.
- It is also used in a guitar and other musical instruments to give loud notes.
- Hearing occurs when eardrum resonates to sound waves hitting it.
- Soldiers do not march in time across bridges to avoid resonance and large amplitude vibrations. Failure to do so caused the loss of over two hundred French infantry men in 1850.
- An opera singer claims to be able to break a wine glass by loudly singing a note of a particular frequency.

End of unit three exercises

1) State what is meant by damping.

2) Describe examples of damped oscillations.

3) State what is meant by natural frequency of vibration and forced oscillations.

4) Describe graphically the variation with forced frequency of the amplitude of vibration of an object close to its natural frequency of vibration.

5) Explain what is meant by resonance.

6) Describe examples of resonance where the effect is useful and where it should be avoided.

7) a) Sketch graphs to show light damping, over-damping and critical damping.
   a) Explain why a car shock absorber needs to be a critically damped system rather than an over- damped system
8) Define the following terms:
   i) Simple harmonic motion.
   ii) Damped oscillation.
   iii) Forced oscillation
9) A block of wood of mass 0.25 kg is attached to one end of a spring of constant stiffness

100 Nm–1 The block can oscillate horizontally on a frictionless surface, the other end of the spring being fixed.

a) Calculate the maximum elastic potential energy of the system for a horizontal oscillation of amplitude 0.20 m.

b) How does the kinetic energy of the mass relate to the elastic potential energy?

c) Calculate the maximum speed of the block.

UNIT IV: PROPAGATION OF MECHANICAL WAVES

1. Concept of wave

A wave is a disturbance in the transport of energy from one point to another in a medium or the disturbances that propagate energy through a medium without transport of matter.

The characteristics of waves are the followings:

- Transport of energy
- They do this without a net motion of matter
- They all involve oscillations that are simple harmonic motion.

2. Types of waves

Waves can be categorized according to:

- ***The particle motion relative to the direction of energy propagation***

i. ***Longitudinal waves***: the orientation of particle motion is parallel to the direction of energy propagation; i.e. vibrations are parallel to the wave motion so if the wave is travelling horizontally the particles will be compressed closer together horizontally or expanded horizontally as they go along. Example sound through air and some earthquake waves.
ii. ***Transverse waves***: the orientation of particle motion is perpendicular to the direction of energy propagation; i.e. Vibrations are perpendicular to the wave motion so if the wave is travelling horizontally, the vibrations will be up and down. Examples: light, a stretched rope or trampoline.
iii. ***Surface waves***: they travel in circular motion; example is water waves.

- ***The types of matter they are able to travel through***

i. ***Electromagnetic waves***: they do not need the support to travel; they can travel in vacuum. Examples are light, microwaves, radio waves…

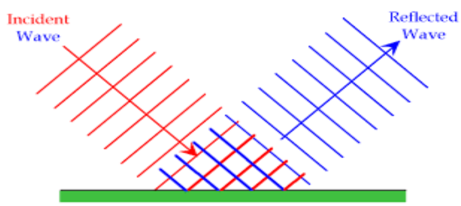
ii. ***Physical (mechanical) waves***: they need a material as support to travel through. Mechanical longitudinal waves can pass through liquid; example is water wave sound waves, earthquakes (seismic waves) and mechanical transverse waves require a solid to travel through it; example: vibrations in a stretched rope.

iii. ***Matter waves:***These waves are associated with electrons, protons, and other fundamental particles, and even atoms and molecules. Because we commonly think of these particles as constituting matter, such waves are called matter waves.However, for specific examples we shall refer to mechanical waves.

3. Properties of mechanical waves

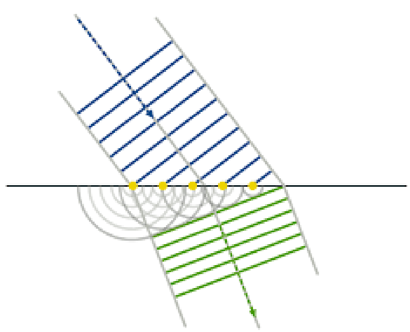
The properties of mechanical waves are reflection, refraction, diffraction and interference

Reflection: when a wave hits a barrier the wave will be bounced back (reflected). If it hits the barrier at an angle then the angle of reflection will be equal to the angle of incidence. The example is the echoes which are caused by the reflection of sound waves.

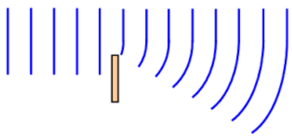


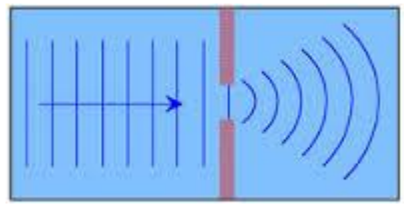
Refraction: when a wave moves from one medium into another, it will either speed up or slow down. For example, a wave going along a rope will speed up if the rope becomes thinner. When a wave speeds up, the wave fronts spread out, the wavelength gets larger. **Note that in both cases, the same number of waves will pass you per second, the wavelength may have changed but the frequency has not.**

If a wave enters a new medium at an angle then the wave fronts also change direction. The amount that the wave is bent by depends on the change in speed. Water waves are slower in shallower water than in deeper water, **so water waves will refract** when the depth changes i.e. when waves slow down, their wavelength gets shorter because the speed is directly proportional to the wavelength, as $v = \lambda f$.

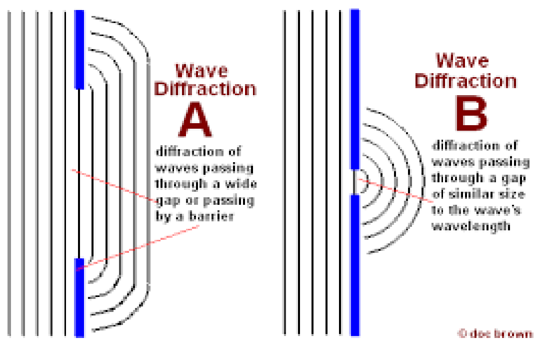


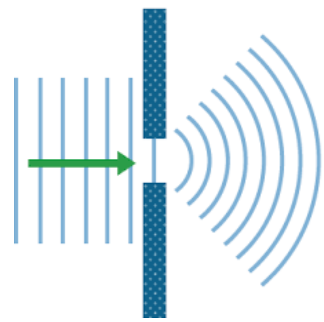
Diffraction: wavefronts change shape when they pass the edge of an obstacle or go through a gap. Diffraction is strong when the width of the gap is similar in size to the wavelength of the waves.





For large aperture For small aperture

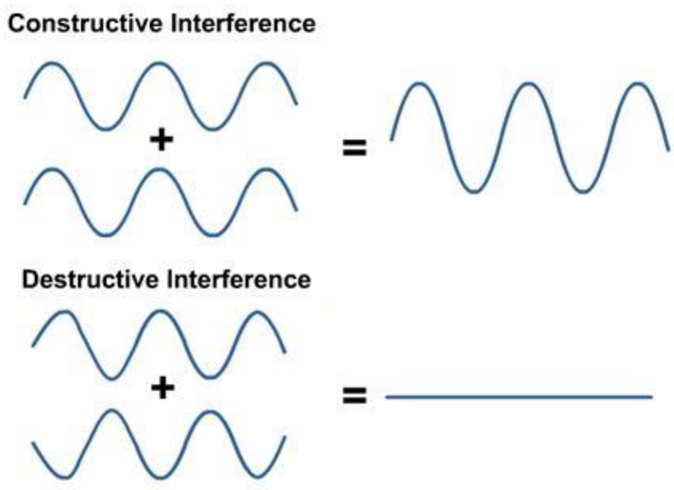




There are two types of diffraction: Fresnel's diffraction and Fraunhofer diffraction.
In Fresnel's diffraction, either the source of waves or screen on which diffraction is observed or both are at finite distances from the obstacle that cause diffraction.
In Fraunhofer diffraction, the sources of waves and the screen on which diffraction is observed are effectively at infinite distances from the obstacle. This phenomenon is practically complicated but theoretically understood. To obtain waves to or from infinite source in laboratory, biconvex lenses are used.

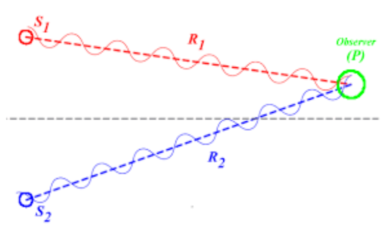
Interference: is a phenomenon in which two waves of the same frequency, same amplitude with a constant phase difference superimpose to form a resultant wave of greater or lower amplitude.

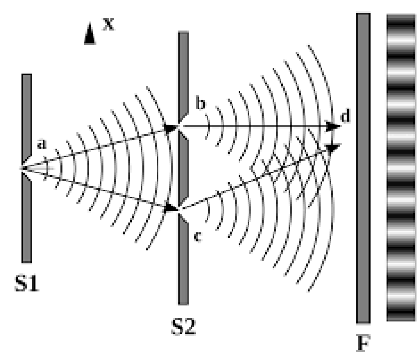
Interference usually refers to the interaction of waves that are correlated or coherent with each other, either because they come from the same source or because they have the same or nearly the same frequency. Examples are sounds waves from two distant loudspeakers. The points with maximum vibrations interfere to form constructive interference while those of zero amplitude will form destructive interference (silent points).



*If two waves having the same frequency and amplitude are in phase the resultant wave when they combine has the same frequency as the individual waves but twice their amplitude; this resultant is called the **constructive interference**.

When two waves of the same frequency and amplitude are out of phase (the phase difference is 180^0) the result when they combine is complete cancellation called **destructive interference**.





Polarization: transverse waves can be forced to oscillate in one fixed direction only; this is polarization of waves.

4. **Progressive waves**

A progressive wave is a wave which spread out energy from the source vibration into the surrounding space. Example is when a ripple is made on the surface of puddle. As the energy spreads out, so its intensity decreases.

A progressive wave is described as follow:

a. Every particle of medium executes periodic motion
b. The amplitude of each particle of the medium is same, but there exists phase difference between them
c. The distance between 2 successive crests of a transverse wave and distance between a compression and rarefaction is a wavelength
d. The changes in pressure and density of the medium are similar in case of progressive waves
e. In a progressive wave, the particle of the medium wave attain a stationary position
f. The equation of a progressive wave is $y = A\sin\frac{2\pi}{\lambda}(vt - x)$

Examples of mechanical waves

Mechanical waves, being **progressive** and **stationary**, are seen in different forms as :

Sound waves are longitudinal waves; they travel fastest in solids, slower in liquids and slowest in gases.

Water waves are combination of both transverse and longitudinal waves. These waves are periodic disturbances that move away from the source and carry energy as they go.

Ocean waves are longitudinal waves that are observed moving through the bulk of liquids, such as our oceans

Earthquake waves occur when elastic energy is accumulated slowly within the Earth's crust (as result of plate motions) and then released suddenly along fractures in the crust called faults. Earthquake waves are also called ***seismic waves*** and actually travel as both transverse and longitudinal waves. The P waves (Primary waves or compressional waves) in an Earthquake are examples of longitudinal waves.

Body waves are of two types: compressional or primary waves, which are longitudinal in nature and shear or secondary waves, which are transverse in nature. P and S waves are called 'body waves' because they can travel through the interior of a body, such as Earth's inner layers, from the focus of an earthquake to distant points on the surface. The Earth's molten core is only travelled by compressional waves.

Surface waves occur at or near the boundary between two media, a transverse wave and a longitudinal wave can combine to form a surface wave. Examples of surface waves are a type of seism wave formed as a result of an earthquake and water waves.

5. Different forms of the equation of a progressive wave

$y = A\sin \omega t$

$y = A\sin(\omega t - \phi)$; the wave is travelling from left to right (positive direction).

$y = A\sin(\omega t - kx)$

$y = A\sin(2\pi f t - \phi)$

$y = A\sin\left(\omega t - \frac{2\pi x}{\lambda}\right)$ but $\omega = \frac{2\pi}{T}$

$y = A\sin\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right)$ but $v = \frac{\lambda}{T}$ or $T = \frac{\lambda}{v}$

$y = A\sin 2\pi\left(\frac{vt}{\lambda} - \frac{x}{\lambda}\right)$

Or $y = A\sin \frac{2\pi}{\lambda}(vt - x)$

If the wave is travelling from the right to left (negative direction) the wave is written as follow: $y = A\sin \frac{2\pi}{\lambda}(vt + x)$ i.e. $y = A\sin(kvt + kx)$ and vice versa.

6. Superposition of progressive waves

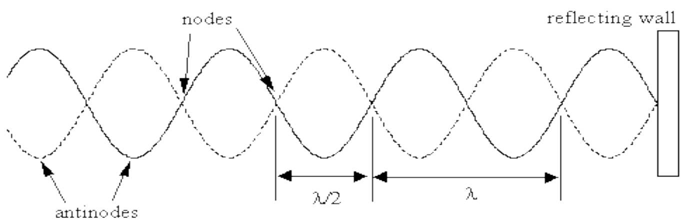
When two identical progressive waves travelling in a medium with same velocity but in opposite directions along the same straight line are superposed, then give rise to a system of alternative rarefaction and compression that cannot move in any direction of medium. This resultant wave is called **stationary or standing wave.**

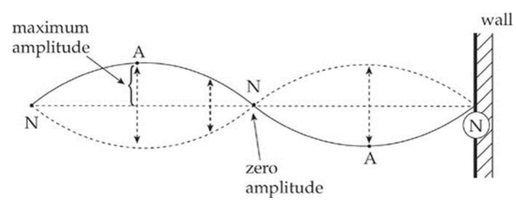
They are called stationary because there is no flow of energy along the waves. There are certain points, half a wavelength apart, which are permanently at rest, are known as **nodes** and there are some other points midway between the nodes where the displacement is maximum, known as **antinodes**.

7. Standing (Stationary) waves

A standing wave is a vibration of a system in which some particular points remain fixed while others between them vibrate with the maximum amplitude. The positions of their peaks and troughs do not move. Some parts of the string, for example, will vibrate while other parts (e.g. ends) do not. A standing wave pattern is a vibrational pattern created within a medium when the vibrational frequency of the source causes ***reflected waves*** from one end of the medium to interfere with incident waves from the source.

Standing waves result in the superposition of two waves of the same amplitude and frequency travelling at the same speed in opposite directions. The wave patterns appear to be standing due to the medium vibrating at specific frequencies called ***harmonic frequencies*** or merely ***harmonics***. The harmonic corresponds to the loop and each harmonic equal in length to a half-wavelength $\frac{\lambda}{2}$. A standing wave is a series of nodes and antinodes.





Nodes are points vibrating with an amplitude, which is equal to zero while the Antinodes are the points vibrating with a maximum amplitude.

a. Characteristics of Stationary Waves

A standing wave is described as follows:

1. Stationary waves are produced when two identical waves travelling along the same straight line but in opposite direction are superposed.
2. Crests and troughs do not progress through the medium but simply appear or disappear at the same place alternatively.
3. All the particles, except those at the nodes, follow simple harmonic motion.
4. The amplitude of the oscillation is zero at nodes and maximum at antinodes.
5. The distance between two successive antinodes or two nodes is equal to half of wavelength i.e. the distance between 3 successive nodes or antinodes is called **a wavelength**.

5. The particle between two successive nodes are in the same phase of vibration while the particles on opposite sides of a node are in opposite phase of vibration.
6. Stationary waves can be produced both by longitudinal waves and transverse waves.

7. All the particles pass through their mean positions or reach their outermost positions simultaneously twice in a periodic time.
8. There is no advancement of the wave and no flow of energy in any direction.
9. In case of standing wave, pressure and density remains almost unchanged at the nodes, while the changes are minimum at the antinodes.
10. The equation of a stationary wave is $y = A\sin\frac{2\pi}{\lambda}(vt)$

b. **Analytical treatment of stationary waves and their properties**

Consider two plane progressive waves-one travelling along positive *X*-axis and another along negative *X*-axis with amplitude a, wave velocity v and wavelength λ. So, the superposing progressive waves are $y_1 = a\sin\frac{2\pi}{\lambda}(vt - x)$ and $y_2 = a\sin\frac{2\pi}{\lambda}(vt + x)$.

By the principle of superposition, the resultant displacement of a particle at x at time t will be

$$y = y_1 + y_2 = a\sin\frac{2\pi}{\lambda}(vt - x) + a\sin\frac{2\pi}{\lambda}(vt + x)$$

$$y = 2a\sin\frac{2\pi}{\lambda}vt\cos\frac{2\pi}{\lambda}x \quad (1)$$

$$y = A\sin\frac{2\pi vt}{\lambda}$$

Hence $A = 2a\cos\frac{2\pi}{\lambda}x$

This equation represents a simple harmonic motion of same wavelength of the superposed wave but not of same amplitude. Moreover, the amplitude $A = 2a\cos\frac{2\pi}{\lambda}x$ is not a constant. For different values of *x*, A will have different values. Equation of motion given by equation (1) is not a progressive motion since its phase does not contain any term like $(vt - x)$ or $(vt + x)$. So, equation (1) represents a stationary wave.

c. **Standing wave equation (free end).**

The reflected wave will have the same amplitude, velocity and wave length.

$y_1 = a\sin(\omega t - kx)$ (to the right) incident wave. $y_2 = a\sin(\omega t + kx)$ (to the left) reflected wave.

Using superposition $y = y_1 + y_2 = a\sin(\omega t - kx) + a\sin(\omega t + kx)$

$y = 2a\cos(kx)\sin(\omega t)$.

Position of nodes and antinodes

For a free end (open end), the oscillation amplitude varies with location according to $\cos\frac{2\pi x}{\lambda}$ where $\frac{2\pi}{\lambda}=k$.

At the nodes: $y=0$ always $\cos\frac{2\pi x}{\lambda}=0$ i.e. $\frac{2\pi x}{\lambda}=n\frac{\pi}{2}$ with $n\in 2N+1$ (n is an odd number).

$$x=\frac{n\lambda}{4}$$

The distance between the nodes is equal to $\frac{\lambda}{2}$.

At the antinodes: y reaches maximum amplitude $2a$. This happens when $\cos\frac{2\pi x}{\lambda}=\pm 1$ i.e. $\frac{2\pi x}{\lambda}=m\pi$ with $m\in N$ and $x=\frac{m\lambda}{2}$ (m is a natural number). The distance between the antinodes is $\frac{\lambda}{2}$.

For two fixed end (closed end), the oscillation amplitude varies with location according to $\sin kx=\sin\frac{2\pi x}{\lambda}$

since $k=\frac{2\pi}{\lambda}$.

At the nodes: $y=0$ always. $\sin\frac{2\pi x}{\lambda}=0$ i.e. $\frac{2\pi x}{\lambda}=n\pi$ with $n\in N$ and $x=\frac{n\lambda}{2}$.

At the antinodes: $\sin\frac{2\pi x}{\lambda}=\pm 1$, therefore $\frac{2\pi x}{\lambda}=n\left(\frac{\pi}{2}\right)$ with $n\in(2N+1)$ and $x=\frac{n\lambda}{4}$

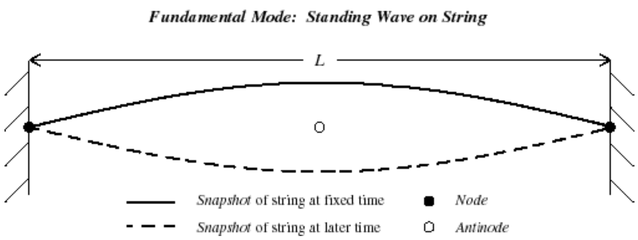
The distance between one node and next antinode is $\frac{\lambda}{4}$.

d. Standing waves in vibrating strings

Standing waves are produced on a string when equal waves travel in opposite directions. When the proper conditions are met, the interference between the traveling waves causes the string to move up and down in segments. This segment vibration gives no appearance of motion along the length of the string. The phenomenon is called **a standing wave or stationary wave** and corresponds to a resonant vibration of the string. The velocity, v, of a wave on a stretched string is given by: $v=\sqrt{\frac{T}{m}}$ (1) where T is the stretching force (tension) and m is the mass per unit length of the string. The same equation can be written as follow $v=\sqrt{\frac{T}{\mu}}$ where $\mu=\frac{m}{l}$ or simply $v=\sqrt{\frac{Tl}{m}}$. If the general wave equation: $v=f\lambda$ (2) is combined with the above equation, the frequency of the vibrator is given by the relation: $f=\frac{1}{\lambda}\sqrt{\frac{T}{m}}$ (3). TO determine the quantity $\frac{T}{\lambda^2}$ and you will be given m, so the frequency squared can be obtained from: $f^2 = \frac{\frac{T}{\lambda^2}}{m}$ i.e. $f^2 = \frac{T}{\lambda^2 m}$.

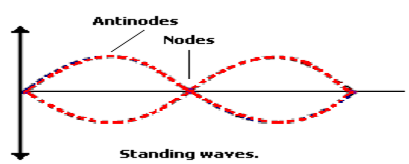
Experiment to produce standing waves in a vibrating string

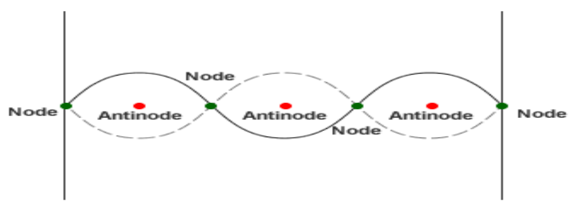
The simplest standing wave pattern that could be produced within a string is one that has points of no displacement (nodes) at the two ends of the string and one point of maximum displacement (antinode) in the middle i.e. it has one loop.



This is the first harmonic.

As in all standing wave patterns, every node is separated by an antinode by $\frac{\lambda}{4}$ (a quarter of the wavelength). This pattern with three nodes and two antinodes (two loops) is referred to as the second harmonic; that one with three antinodes (loops) and four nodes is the third harmonic; and they are depicted in the figure shown below:





So the following table gives information about the harmonics, number of nodes and antinodes and the length of the string; the location of nodes and antinodes when the ends are closed.

Harmonic	Number of nodes	Number of antinodes	Length of the string
1st	2	1	$L = \frac{1}{2}\lambda$
2nd	3	2	$L = \lambda$
3rd	4	3	$L = \frac{3}{2}\lambda$
4th	5	4	$L = 2\lambda$
5th	6	5	$L = \frac{5}{2}\lambda$
nth	n+1	N	$L = \frac{n}{2}\lambda$

Examples of stationary waves in daily life are produced in **musical instruments**. The examples include waves formed on vibrating strings of guitars and violins and also vibrating air column in pipe instruments such as organs and flutes. They are also formed in air bottles when air is blown over the open top of the bottle.

Applications of waves

Waves are used:

- In radar, broadcasting and radio communication
- In (MRI) magnetic resonance imaging in hospitals
- In radio communication which forms an integral part of wireless communication
- In speaking (vocal cords), hearing and in all musical instruments (production of sounds).

End unit four exercises

1) **The speed of sound in air is a bit over 300 m/s, and the speed of light in air is about 300,000,000 m/s. Suppose we make a sound wave and a light wave that both have a wavelength of 3 meters.**

   What is the ratio of the frequency of the light wave to that of the sound wave?

   (A) About 1,000,000

   (B) About 0.000,001

   (C) About 1000

2) **Which of the following equation describes a harmonic wave moving in the <u>negative x direction</u>**

   (A) $D(x,t) = A \sin (k x - wt)$

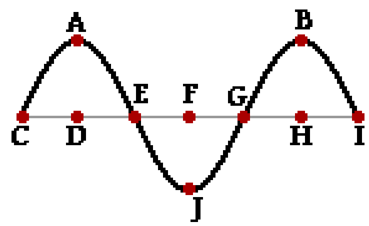
   (B) $D(x,t) = A \cos (k x + wt)$

   (C) $D(x,t) = A \cos (-k x + wt)$

3) f the particles of the medium are vibrating to and fro in the same direction of energy transport, then the wave is a ____ wave.

A. longitudinal b. sound c. standing d. transverse

4) A transverse wave is traveling through a medium. See diagram below. The particles of the medium are vibrating _____.
5)



a. parallel to the line joining AD.

b. along the line joining CI.

c. perpendicular to the line joining AD.

d. at various angles to the line CI.

e. along the curve CAEJGBI.

6) A sound wave has a frequency of 192Hz and travels the length of a football field, 91.4m in 0.271s.
   i) What is the speed of the wave?
   ii) What is the wavelength of the wave?
   iii) What is the period of the wave?
   iv) If the frequency was changed to 442Hz, what would be the new wavelength and period?
7) a) Determine the direction of propagation of a plane progressive wave represented by the equation $y = 0.5\sin(100\pi t - \frac{20\pi x}{17})$ where y is the displacement in millimeters, t is in seconds and x is the distance from a fixed origin O in meters
   b) Find the frequency of this wave.